1. Show that any strongly connected undirected graph with n vertices and $(n-1)$ edges is a tree.

2. We know that the strongly connect components in any directed graph form a partition of vertices in the graph. So, the strongly connected components in a given graph can be represented as a partition of vertices. Consider the directed graphs G_1 and G_2 below and answer the questions that follow:
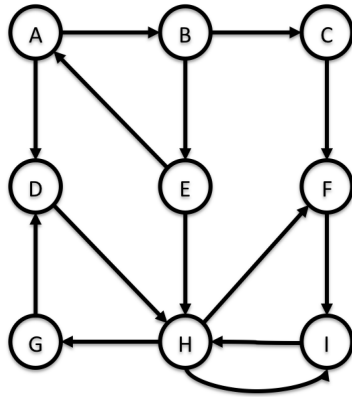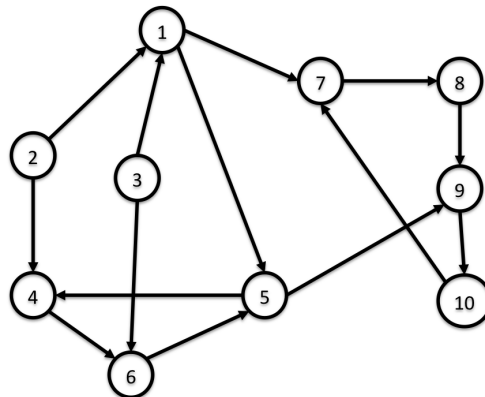
(a) Graph G_1 (b) Graph G_2

(a) Give the strongly connected components of graph G_1.

(a) _____

(b) Give the strongly connected components of graph G_2.

(b) _____

3. Given a directed graph $G = (V, E)$ and an edge $(u, v) \in E$, you want to determine if G has a cycle that contains this edge (u, v). Design an algorithm for this problem. Give pseudocode, discuss running time, and give proof of correctness.

4. You are given a directed acyclic graph $G = (V, E)$ in which each node $u \in V$ has an associated *price*, denoted by $price(u)$, which is a positive integer. The *cost* of a node u, denoted by $cost(u)$, is defined to be the price of the cheapest node reachable from u (including u itself). Design an algorithm that computes $cost(u)$ for all $u \in V$. Give pseudocode, discuss running time, and give proof of correctness.