COL863: Quantum Computation and Information

Ragesh Jaiswal, CSE, IIT Delhi

Quantum Computation: Basic Quantum Algorithms

- We will see some basic quantum algorithms that were the precursor to the more popular algorithms such as Factoring. The main ideas were developed in these simple algorithms.
  - Bernstein-Vazirani
  - Simon's problem
- While discussing these algorithms we will try to argue why quantum algorithms have an advantage compared to classical ones.

Problem

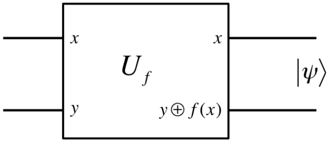
Given a function $f : \{0,1\}^n \rightarrow \{0,1\}$ such that for every $x \in \{0,1\}^n$, $f(x) = (a \cdot x)$, determine a. Here $(a \cdot x)$ denotes the dot product of bit vectors a and x.

- Question: In the classical setting, how many classical queries to the function f will be needed to determine a?

Problem

Given a function $f : \{0,1\}^n \rightarrow \{0,1\}$ such that for every $x \in \{0,1\}^n$, $f(x) = (a \cdot x)$, determine a. Here $(a \cdot x)$ denotes the dot product of bit vectors a and x.

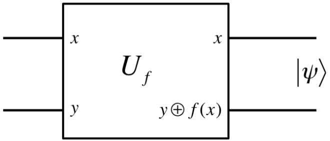
- Question: In the classical setting, how many classical queries to the function f will be needed to determine a? n queries
  - Since each query reveals at most one bit of a.
- Question: Suppose the unitary transformation below is available to us (as in Deutsch-Jozsa). How many invocations of this gate will be required within the quantum circuit to determine a?



Problem

Given a function $f : \{0,1\}^n \rightarrow \{0,1\}$ such that for every $x \in \{0,1\}^n$, $f(x) = (a \cdot x)$, determine a. Here $(a \cdot x)$ denotes the dot product of bit vectors a and x.

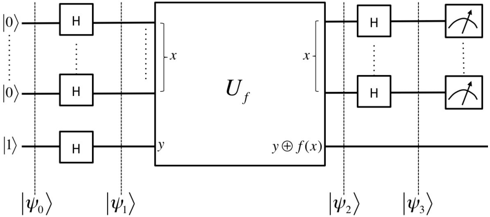
- Question: In the classical setting, how many classical queries to the function f will be needed to determine a? n queries
  - Since each query reveals at most one bit of a.
- Question: Suppose the unitary transformation below is available to us (as in Deutsch-Jozsa). How many invocations of this gate will be required within the quantum circuit to determine a? One!



Problem

Given a function $f : \{0,1\}^n \to \{0,1\}$ such that for every $x \in \{0,1\}^n$, $f(x) = (a \cdot x)$, determine a. Here $(a \cdot x)$ denotes the dot product of bit vectors a and x.

- Question: In the classical setting, how many classical queries to the function f will be needed to determine a? n queries
- Question: Suppose the unitary transformation below is available to us (as in Deutsch-Jozsa). How many invocations of this gate will be required within the quantum circuit to determine a? One!
    - The same circuit as in Deutsch-Jozsa works!
    - Question: What will be the measurement output of the circuit below?



Problem

Given a function $f : \{0,1\}^n \rightarrow \{0,1\}$ such that for every $x \in \{0,1\}^n$, $f(x) = (a \cdot x)$, determine a. Here $(a \cdot x)$ denotes the dot product of bit vectors a and x.

- Question: In the classical setting, how many classical queries to the function f will be needed to determine a? n queries
- Question: Suppose the unitary transformation below is available to us (as in Deutsch-Jozsa). How many invocations of this gate will be required within the quantum circuit to determine a? One!
- Does this really show that quantum computers are more powerful?
  - The function f is only accessible as a black-box in the classical setting.
  - There may be a classical algorithm that figures out a if the circuit implementing f is accessible.

Problem

Given a function $f : \{0,1\}^n \rightarrow \{0,1\}$ such that for every $x \in \{0,1\}^n$, $f(x) = (a \cdot x)$, determine a. Here $(a \cdot x)$ denotes the dot product of bit vectors a and x.

- Question: In the classical setting, how many classical queries to the function f will be needed to determine a? n queries
- Question: Suppose the unitary transformation below is available to us (as in Deutsch-Jozsa). How many invocations of this gate will be required within the quantum circuit to determine a? One!
- Does this really show that quantum computers are more powerful? Yes and no
- The above shows a gap factor of n. Can we design a similar problem that has super-polynomial gap? Yes using a recursive extension of the above ideas.

Simon's problem

Given a function $f : \{0,1\}^n \rightarrow \{0,1\}^n$ that satisfies the following conditions: (i) $f(x) = f(y) \leftrightarrow x \oplus y = a$, (ii) $a \neq 0...0$. The problem is to determine a.

- Such a function is called a 2-to-1 function.
- Question: How many classical queries to the function f need to be made to find a?

Simon's problem

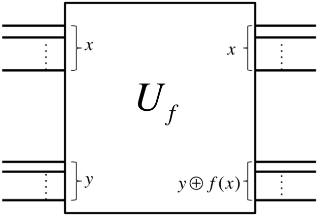
Given a function $f : \{0,1\}^n \rightarrow \{0,1\}^n$ that satisfies the following conditions: (i) $f(x) = f(y) \leftrightarrow x \oplus y = a$, (ii) $a \neq 0...0$. The problem is to determine a.

- Question: How many classical queries to the function f need to be made to find a? $\Theta(2^{n/2})$
  - $O(2^{n/2})$ queries are suffcient using birthday bound.
  - $\Omega(2^{n/2})$ queries are necessary using an information-theoretic argument.

Simon's problem

Given a function $f : \{0,1\}^n \rightarrow \{0,1\}^n$ that satisfies the following conditions: (i) $f(x) = f(y) \leftrightarrow x \oplus y = a$, (ii) $a \neq 0...0$. The problem is to determine a.

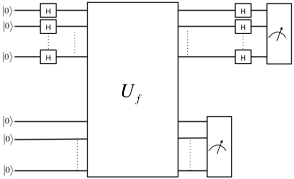
- Question: How many classical queries to the function f need to be made to find a? $\Theta(2^{n/2})$
- Question: Suppose the following gate is available. How many invocations of this gate will be required in the quantum setting to obtain a?



Simon's problem

Given a function $f : \{0,1\}^n \rightarrow \{0,1\}^n$ that satisfies the following conditions: (i) $f(x) = f(y) \leftrightarrow x \oplus y = a$, (ii) $a \neq 0...0$. The problem is to determine a.

- Question: How many classical queries to the function f need to be made to find a? $\Theta(2^{n/2})$
- Question: Suppose the following gate is available. How many invocations of this gate will be required in the quantum setting to obtain a? $\Theta(n)$
  - Running the circuit below $\Theta(n)$ times will be sufficient to determine a.



Simon's problem

Given a function $f : \{0,1\}^n \rightarrow \{0,1\}^n$ that satisfies the following conditions: (i) $f(x) = f(y) \leftrightarrow x \oplus y = a$, (ii) $a \neq 0...0$. The problem is to determine a.

- Question: How many classical queries to the function f need to be made to find a? $\Theta(2^{n/2})$
- Question: Suppose the following gate is available. How many invocations of this gate will be required in the quantum setting to obtain a? $\Theta(n)$
- Does this really show that quantum computers are more powerful? Yes and no

End