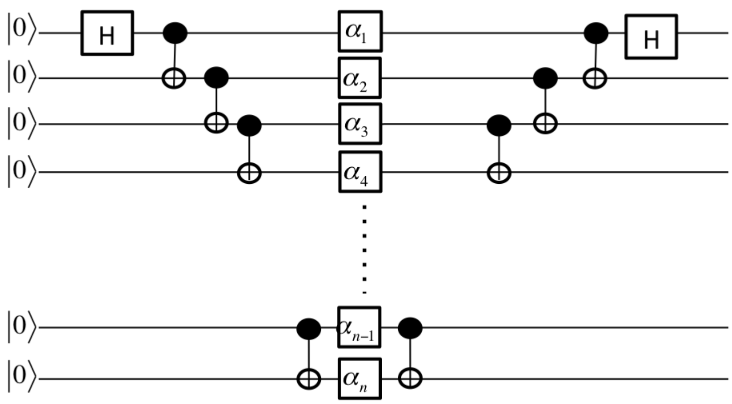
COL863: Quantum Computation and Information
Quiz: 2

1. Draw a quantum circuit on two qubits that copies the $\left|+\right\rangle, \left|-\right\rangle$ state. That is, the circuit should have the following behaviour:

Input	Output
$\left|+\right\rangle\left|0\right\rangle$	$\left|+\right\rangle\left|+\right\rangle$
$\left|-\right\rangle\left|0\right\rangle$	$\left|-\right\rangle\left|-\right\rangle$

2. Consider the circuit below:



Here α_i denotes the operation $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\alpha_i} \end{bmatrix}$. Suppose at the end the measurement is taken in the computational basis. What is the probability of seeing $\left|00...0\right\rangle$?

3. Show that $R_x(\theta)$ defined as $e^{-i\theta X/2}$ is indeed $\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}X$. (*See the last linear-algebra slide for the meaning of* $e^{-i\theta X/2}$).

4. You are given a function $f : \{0,1\}^n \rightarrow \{0,1\}$ which has the following property:
- There exists a string $s \in \{0,1\}^n$ such that for every $x \in \{0,1\}^n, f(x) = (s \cdot x)$, where $(\cdot)$ indicates the dot product modulo 2.

Your goal in this problem is to find the string s. Note that in the classical setting you can find s by making n queries to the function. We would like to do this using much lesser queries in the Quantum setting. Describe how you can use the quantum circuit for the Deutsch-Jozsa problem to solve this problem.