COL863: Quantum Computation and Information
Homework: 2 (*This is for practice. You need not submit.*)

1. Exercises from the book: 2.2, 2.3, 2.4, 2.7, 2.8, 2.9, 2.10, 2.11, 2.12, 2.13, 2.14, 2.15, 2.16, 2.17, 2.18, 2.19, 2.20, 2.22, 2.23, 2.24, 2.25, 2.26, 2.27, 2.28, 2.29, 2.30, 2.31, 2.32, 2.33, 2.34.

2. A *P-matrix* is a matrix $\Pi \in \mathbb{C}^{n\times n}$ such that $\Pi^2 = \Pi$. Answer the following question:

   State true or false with reasons: For any Hermitian matrix $\Pi \in \mathbb{C}^{n\times n}$, Π is a *P-matrix* if and only if $\Pi = \sum_{i=1}^{k} |v_i\rangle \langle v_i|$ for some orthonormal vectors $|v_1\rangle, |v_2\rangle, ..., |v_k\rangle \in \mathbb{C}^n$.

3. An operator M on a finite dimensional vector space V with inner products is said to be *norm preserving* if for every $|w\rangle \in V$, $|| \, |w\rangle \, || = ||M \, |w\rangle \, ||$. Answer the following question.

   - Let $|v_1\rangle, ..., |v_n\rangle$ be an orthonormal basis for V. What conditions on numbers $a_1, ..., a_n \in \mathbb{C}$ are necessary and sufficient for $M \equiv \sum_{i=1}^{n} a_i |v_i\rangle \langle v_i|$ to be norm preserving? Give reasons.

4. Show that any positive operator on a finite dimensional inner product space is necessarily a Hermitian.