COL863: Quantum Computation and Information
Homework: 1 (*This is for practice. You need not submit.*)

1. Can the following two-qubit state $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ be represented as $(\alpha\,|0\rangle+\beta\,|1\rangle)(\alpha'\,|0\rangle+\beta'\,|1\rangle)$?

2. Can there exist a single qubit gate with the following truth table? Give reasons.

Input	Output
$\|0\rangle$	$\frac{\sqrt{3}}{2}\|0\rangle + \frac{1}{2}\|1\rangle$
$\|1\rangle$	$\frac{1}{2}\|0\rangle + \frac{\sqrt{3}}{2}\|1\rangle$

3. Show that there exist a single qubit gate with the following truth table? Give the matrix representation of such a gate.

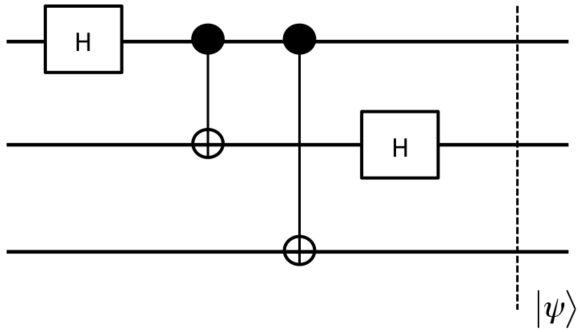
Input	Output
$\|0\rangle$	$\frac{\sqrt{3}}{2}\|0\rangle - \frac{1}{2}\|1\rangle$
$\|1\rangle$	$\frac{1}{2}\|0\rangle + \frac{\sqrt{3}}{2}\|1\rangle$

4. Draw the classical circuit for computing the Boolean function $f : \{0,1\}^2 \rightarrow \{0,1\}$ given by the following truth table.

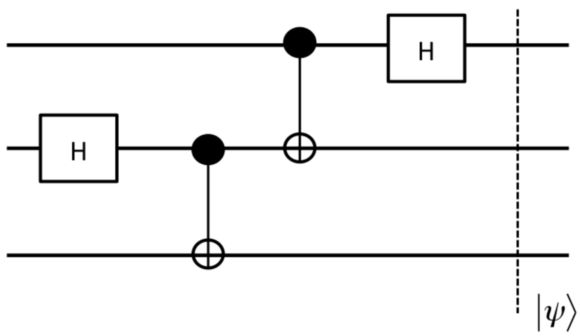
x	$f(x)$
00	1
01	0
10	1
11	0

Give the Quantum analogue of your classical circuit using Toffoli gates.

5. Output $|\psi\rangle$ when the input to the circuit is $|000\rangle$. Output $|\psi\rangle$ when the input is $[\alpha\,|0\rangle + \beta\,|1\rangle]\,|00\rangle$.

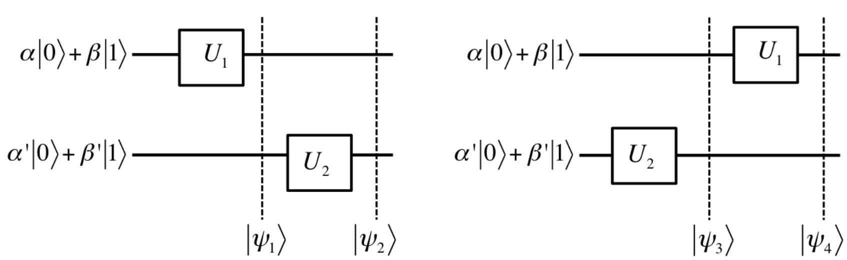


6. Output $|\psi\rangle$ when the input to the circuit is $|000\rangle$. Output $|\psi\rangle$ when the input is $[\alpha|0\rangle + \beta|1\rangle]|00\rangle$.

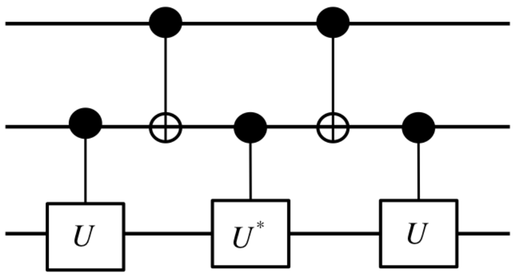


7. Can you use a single qubit as a source of randomness? How?

8. Let the matrix representation of gates U_1 and U_2 be $U_1 = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $U_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Give the states $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle$ in the circuits below.

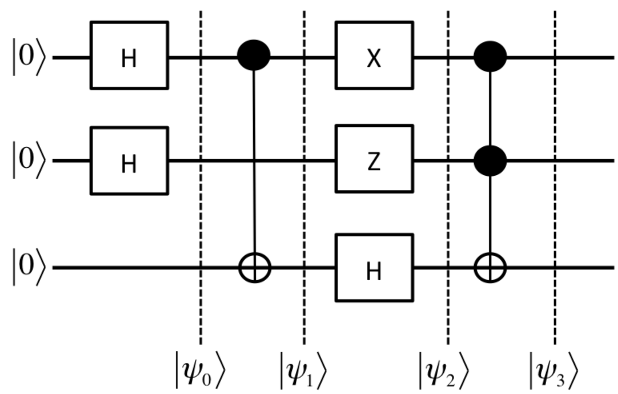


9. What is the input-output behaviour of the following circuit. (*U^* denotes conjugate transpose.*)



Input	Output
$\lvert 00\rangle\lvert\psi\rangle$	
$\lvert 01\rangle\lvert\psi\rangle$	
$\lvert 10\rangle\lvert\psi\rangle$	
$\lvert 11\rangle\lvert\psi\rangle$	

10. Give the the intermediate states $|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle$ of the 3-qubit circuit given below. Show your calculations.



11. Suppose you have two qubits in the bell state $\frac{|01\rangle-|10\rangle}{\sqrt{2}}$ and you apply the teleportation protocol to the first qubit. What is the result? (*Please try giving an appropriate interpretation for your calculations.*)