

1. You are given a weighted, directed graph where the weight of a node u denotes the cost $c(u) > 0$ of the node. The price $p(u)$ of a node u is the cost of the cheapest node that is reachable from u . Design an algorithm that computes the price of all nodes in a given graph. Give proof of correctness and running time analysis.
2. Design an algorithm for finding the in-degrees of all vertices in a given directed graph $G = (V, E)$.
3. Consider the following algorithm for topologically sorting the vertices of a given DAG $G = (V, E)$.

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NewTopoSort( $G$ )
- Initialise an empty list  $L$ 
- While  $G$  has at least one node:
  - Let  $v$  be a source node in  $G$ 
  - Append  $v$  to  $L$ 
  - Remove vertex  $v$  and its out-going neighbours edges from  $G$  to obtain graph  $G'$ 
  -  $G \leftarrow G'$ 
- return( $L$ )
```

Argue that the above algorithm produces a topological ordering of vertices for any DAG. Describe an implementation of the algorithm that has linear running time.