

1. Prove or disprove: There are an even number of odd-degree vertices in any undirected graph.
(*Degree of a vertex is the number of edges incident on that vertex.*)

2. In the lectures, we learnt that the two main graph representations are adjacency list and adjacency matrix. Design algorithms for the following tasks:
 - (a) Given a graph $G = (V, E)$ as input in the adjacency matrix representation, output the adjacency list representation of G .
 - (b) Given a graph $G = (V, E)$ as input in the adjacency list representation, output the adjacency matrix representation of G .

Discuss running time of your algorithms.

3. The reverse of a directed graph $G = (V, E)$ is another directed graph $G^R = (V, E^R)$ on the same vertex set but with all the edges reversed. Design an algorithm that outputs the adjacency list of the reverse of a given graph G . G given as input in adjacency list format. Discuss running time.

4. An undirected graph is said to be *bipartite* iff its vertices can be partitioned into two sets such that there are no edges between any two vertices in the same partition.
Design an algorithm to determine if a given undirected graph is bipartite. Give proof of correctness and running time analysis.