

There are 2 questions for a total of 10 points.

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1. Apply the Master theorem and give the solution for the following recurrence relations in big-O notation. Explanation is not required.

(a) (1 point)  $T(n) = 2 \cdot T(n/2) + O(1); T(1) = O(1)$

(b) (1 point)  $T(n) = 2 \cdot T(n/2) + O(n); T(1) = O(1)$

(c) (1 point)  $T(n) = 2 \cdot T(n/2) + O(n^3); T(1) = O(1)$

2. (7 points) You are given a bit-array  $A[1..n]$  (i.e.,  $A[i] \in \{0, 1\}$  for each  $i$ ) and told that this is a “0-to-1” bit-array. This means that for some (unknown) index  $1 \leq j < n$ ,  $A[1], \dots, A[j]$  are all 0's and  $A[j + 1], \dots, A[n]$  are all 1's. The index  $j$  for such an array is called the transition index.

Design an algorithm for finding the transition index for a given 0-to-1 bit-array. The input to your algorithm is an array  $A$  and the size  $n$  of the array  $A$ . Give a running time analysis for your algorithm.