Name: ____________________

Entry number: ____________________

There are 3 questions for a total of 20 points.

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1. (6 points) Find the eigenvalues and corresponding eigenvectors for the following matrices (corresponding to single qubit gates):

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

2. (6 points) Suppose Bob is given a quantum state chosen from a set $|\psi_1\rangle, |\psi_2\rangle, ..., |\psi_m\rangle$ of linearly independent states. Construct a POVM $\{E_1, E_2, ..., E_{m+1}\}$ such that if outcome E_i occurs, $1 \leq i \leq m$, then Bob knows with certainty that he was given the state $|\psi_i\rangle$.

3. (8 points) Suppose you have two qubits in the bell state $\frac{|01\rangle - |10\rangle}{\sqrt{2}}$ and you apply the teleportation protocol to the first qubit. What is the result? (*Please try giving an appropriate interpretation for your calculations.*)

Space for rough work