CSL202: Discrete Mathematical Structures

Ragesh Jaiswal, CSE, IIT Delhi

Cardinality of Sets

Definition

The sets A and B have the same cardinality if there is a one-to-one correspondence from A to B. When A and B have the same cardinality, we write $|A| = |B|$.

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If there is a one-to-one function from A to B, the cardinality of A is less than or the same as the cardinality of B and we write $|A| \leq |B|$. The cardinality of A is less than the cardinality of B, written as $|A| < |B|$, if there is an injection but no surjection from A to B.

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Theorem

Let S be a set. Then $|S| < \mathcal{P}(S)$.

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- Show that the set of odd positive integers is a countable set.

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- Show that the set of all integers is countable.

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- Show that the set of positive rational numbers is countable.

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- An infinite set is countable if and only if it is possible to list the elements of the set in a sequence (indexed by the positive integers).
- Show that the set of all integers is countable.
- Show that the set of positive rational numbers is countable.
- Show that the set of real number is an uncountable set.

- Which of the following statements true:
  - Every integer has a finite size description in decimal.
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- Where does the diagonalization argument fail in case of integers?

- If A and B are countable sets, then $A \cup B$ is also countable.

Theorem (Schröder-Bernstein theorem)

If there are one-to-one functions f from A to B and g from B to A, then there is a one-to-one correspondence between A and B.

- Show that $|(0,1)| = |(0,1]|$.

End