Name: ______________________________

ID number: ______________________________

There are 4 questions for a total of 10 points.

---

1. (2 points) Show that the following compound proposition is a tautology:

$$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$$

Solution: We will show that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p \equiv T$ by showing a chain of equivalences.

$$\begin{aligned}
(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p &\equiv (\neg q \wedge (\neg p \vee q)) \rightarrow \neg p \qquad \text{(using } p \rightarrow q \equiv \neg p \vee q\text{)}\\
&\equiv ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \rightarrow \neg p \qquad \text{(using distributive law)}\\
&\equiv ((\neg q \wedge \neg p) \vee F) \rightarrow \neg p \qquad \text{(using negation law)}\\
&\equiv (\neg q \wedge \neg p) \rightarrow \neg p \qquad \text{(using identity law)}\\
&\equiv \neg(\neg q \wedge \neg p) \vee \neg p \qquad \text{(using } x \rightarrow y \equiv \neg x \vee y\text{)}\\
&\equiv q \vee p \vee \neg p \qquad \text{(using De Morgan's law)}\\
&\equiv q \vee T \qquad \text{(using negation law)}\\
&\equiv T \qquad \text{(using identity law)}
\end{aligned}$$

You could have also shown this using the truth table.

2. (2 points) Show that $(\neg p \rightarrow (q \rightarrow r))$ and $(q \rightarrow p \vee r)$ are logically equivalent.

Solution: We show equivalence of these compound propositions using the following chain of equivalences.

$$\begin{aligned}
(\neg p \rightarrow (q \rightarrow r)) &\equiv (\neg p \rightarrow (\neg q \vee r)) \qquad \text{(using } x \rightarrow y \equiv \neg x \vee y\text{)}\\
&\equiv p \vee \neg q \vee r \qquad \text{(using } x \rightarrow y \equiv \neg x \vee y\text{)}\\
&\equiv \neg q \vee (p \vee r) \qquad \text{(using commutative and distributive laws)}\\
&\equiv q \rightarrow (p \vee r) \qquad \text{(using } x \rightarrow y \equiv \neg x \vee y\text{)}
\end{aligned}$$

3. (2 points) Let $C(p, q, r)$ denote a compound proposition involving simple propositions p, q, and r. Give a compound proposition $C(p, q, r)$ the truth table of which matches the one given below. (*Note that there may be multiple correct answers for this question*)

p	**q**	**r**	$\mathbf{C(p, q, r)}$
T	T	T	T
T	T	F	F
T	F	T	T
F	T	T	F
T	F	F	T
F	T	F	F
F	F	T	T
F	F	F	F

3. $C(p, q, r) = (p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

You might have found a more simplified expression for this problem. The purpose of this exercise was to convey that any boolean function can be written as a compound propositions involving $\neg, \vee, \wedge$. There is a standard method for doing this. Consider all table entries that have T in the last column. For each such table entry, create a conjunction of variables or negations depending on whether the table entry is T or F. Finally, take a disjunction of all such conjunctions. Try proving that the compound proposition created using this way will match the given truth table. (Furthermore, note that since $\vee$ can be written using $\neg$ and $\wedge$. We can write any boolean function using just $\neg$ and $\wedge$.)

4. Let $C(x, y)$ be the statement "x and y have chatted over the internet", where the domain of variables x and y consists of all students in your class. Use quantifiers to express the following two statements:

   (a) (2 points) There are at least two students in your class who have not chatted with the same person in your class.

   (a) $\exists x\ \exists y((x \neq y) \wedge \forall z\ (\neg(C(x, z) \vee \neg C(y, z))))$

   (b) (2 points) There are two students in the class who between them have chatted with everyone else in the class.

   (b) $\exists x\ \exists y\ ((x \neq y) \wedge \forall z\ ((z \neq x) \wedge z \neq y) \rightarrow (C(x, z) \vee C(y, z)))$