COL202: Discrete Mathematical Structures
Tutorial/Homework: 12

1. Discuss Quiz-09 questions.

2. Complete discussion of Tutorial-11 problems in case needed.

3. Let S be the subset of the set of ordered pairs of integers defined recursively by:

   Basis step: $(0,0) \in S$

   Recursive step: If $(a,b) \in S$, then $(a, b+1) \in S$, $(a+1, b+1) \in S$, and $(a+2, b+1) \in S$.

   (a) List the elements of S produced by the first four applications of the recursive definition.

   (b) Use strong induction on the number of applications of the recursive step of the definition to show that $a \leq 2b$ whenever $(a,b) \in S$.

   (c) Use structural induction to show that $a \leq 2b$ whenever $(a,b) \in S$.

4. Six swimmers training together either swam in a race or watched the others swim. At least how many races must have been scheduled if every swimmer had opportunity to watch all of the others?

5. Suppose that 21 girls and 21 boys enter a mathematics competition. Furthermore, suppose that each entrant solves at most six questions, and for every boy-girl pair, there is at least one question that they both solved. Show that there is a question that was solved by at least three girls and at least three boys.