COL202: Discrete Mathematical Structures
Tutorial/Homework: 10

1. Discuss Quiz-08 (in case required).
2. (*This question is from the previous tutorial.*) We will use the following definition of cyclic groups.

   Definition 10.0.1 (Cyclic group) *Let G be a group and let a be any element of this group. Let $\langle a \rangle = \{x \in G | x = a^n$ for some $n \in \mathbb{Z}\}$. The group G is called a cyclic group if there exists an element $a \in G$ such that $G = \langle a \rangle$. In this case, a is called the generator of G.*

   Show that for any prime p, Z_p^* is a cyclic group.
3. Any other questions on the previous three tutorials that may need more discussion.
4. A jigsaw puzzle is put together by successively joining pieces that fit together into blocks. A move is made each time a piece is added to a block, or when two blocks are joined. Use strong induction to prove that no matter how the moves are carried out, exactly $n - 1$ moves are required to assemble a puzzle with n pieces.
5. Find the flaw with the following mathematical induction based "proof" that $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.
   Basis step: $a^0 = 1$ is true by the definition of a^0.

   Inductive step: Assume that $a^j = 1$ for all nonnegative integers j with $j \leq k$. Then note that $a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1$.