COL202: Discrete Mathematical Structures

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Induction and Recursion

- We will only consider simple graphs for this discussion which are graphs that do not have self loops or multi-edges (i.e., multiple edges between a pair of vertices).

Definition (Strongly connected graph)

An undirected graph is called strongly connected iff for every pair of vertices in the graph there is a path between these vertices.

Definition (Tree)

An undirected graph is called a tree iff the graph is strongly connected and does not have any cycles.

Definition (Cycle)

A sequence of vertices $v_1, v_2, ..., v_k$ in an undirected graph is called a cycle iff $k > 3$, $v_1 = v_k$, $v_1, v_2, ..., v_{k-1}$ are distinct, and for every $1 \leq i \leq k-1$, there is an edge between v_i and v_{i+1}.

- Show that: Every tree with n vertices has exactly $(n-1)$ edges.

Definition (Strong induction)

To prove that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function, we complete two steps:

- Basis step: We verify that $P(1)$ is true.
- Inductive step: We show that the conditional statement $[P(1) \wedge P(2) \wedge ... \wedge P(k)] \rightarrow P(k+1)$ is true for all positive integers k.

- Strong induction is sometimes called *second principle of mathematical induction* or *complete induction*.

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- If the inductive step is valid only for integers greater than a particular integer.
  - Basis step: We verify that the propositions $P(b), P(b+1), \ldots, P(b+j)$ are true.
  - Inductive step: We show that $[P(b) \land P(b+1) \land \ldots \land P(k)] \rightarrow P(k+1)$ is true for every integer $k \geq b+j$.

- Show that if n is an integer greater than 1, then n can be written as the product of primes.

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- Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

End