Name: ____________________

Entry number: ____________________

There are 3 questions for a total of 10 points.

---

1. Recall the `Extended-Euclid-GCD` algorithm discussed in class for finding the gcd of positive integers $a \geq b > 0$ and integers x, y such that $ax + by = gcd(a, b)$. The algorithm makes a sequence of recursive calls until the second input becomes 0. For example, the sequence of recursive calls along with the function-call returns for inputs $(2, 1)$ are:

$$\overset{(1,0,1)}{\leftarrow} \texttt{Extended-Euclid-GCD}(2,1) \overset{\overset{(1,1,0)}{\leftarrow}}{\rightarrow} \texttt{Extended-Euclid-GCD}(1,0)$$

   (a) (1 ½ points) Write down the sequence of recursive calls along with function-call returns that are made when the algorithms is executed with inputs $(995, 53)$.

   (b) (½ point) What is the inverse of 53 modulo 995? That is, give a positive integer x such that $53 \cdot x \equiv 1 \ (mod\ 995)$. Write "not applicable" in case no such integer exists.

   (b) ____________

2. <u>State true or false with reasons</u>:

   (a) (1 point) For all positive integers $a \geq b > 0$ there exists *unique* integers x, y such that $ax + by = gcd(a, b)$.

   (a) ____________

   (b) (1 point) Let $m > 2$ be a prime number and let $1 < a < m$ be any integer. Then a has a unique inverse with respect to the operation multiplication modulo m. That is, there is a unique integer $1 < b < m$ such that $ab \equiv 1 \ (mod\ m)$.

   (b) ____________

3. Consider one of the problems in the tutorial sheet related to the possible way of leaving a certain amount of water given two jugs with integer capacities S and L. Recall that you have unlimited source of water and nothing but the two given jugs. Answer the following questions:

   (a) (3 points) Design an algorithm that takes as input three positive integers S, L, and B such that $B < S < L$ and outputs "Not Possible" if it is not possible to leave B litres of water in any of the two jugs and otherwise it outputs the precise way to make sure that one of the jugs has exactly B litres of water.

   (b) (1 point) Execute your algorithm for input $S = 15, L = 21, B = 12$ and write the output below.

   (c) (1 point) Execute your algorithm for input $S = 5, L = 8, B = 3$ and write the output below.

   (d) (1 point) Execute your algorithm for input $S = 21, L = 33, B = 16$ and write the output below.