There are 5 questions for a total of 50 points.

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1. (5 points) Consider a simple triangle graph with 3 vertices. What are the eigenvalues of the normalized Laplacian matrix for this graph? Also give the eigenvectors corresponding to the eigenvalues. Show calculations.

2. (5 points) Argue that the inequality (one side of Cheeger's inequality) $\frac{\lambda_2}{2} \leq \phi(G)$ is tight.
(*Note that all you need to do is to give an example graph with at least* 2 *vertices where* $\frac{\lambda_2}{2} = \phi(G)$.)

3. (20 points) Argue that there is a *family* $\{F\}_n$ of graphs such that for graphs in this family $\phi(G) = O(\sqrt{2\lambda_2})$. $\{F\}_n$ means that there are graphs of all sizes in the family. Note that this implies that the Cheeger's inequality is (approximately) tight even in the other direction.
(*This question will require you to do some self research. You may use the internet for this but please write the solution in your own words.*)

4. (10 points) Show that the VC dimension of triangles in a plane is 7.

5. (10 points) Suppose the instance space $\mathcal{X}$ is $\{0,1\}^d$ and consider the target function c^* that labels an example $\mathbf{x}$ positive if the least index i for which $\mathbf{x}_i = 1$ is odd, else labels $\mathbf{x}$ as negative. Show that the rule can be expressed as a Linear Threshold Function. A linear threshold function is defined by a vector $\mathbf{w}$ and a constant θ. Examples for which $\mathbf{x}^T\mathbf{w} \geq \theta$ are labeled positive and others are labeled negative.