

Name: _____

Entry no.: _____

There are 4 questions for a total of 50 points.

1. Let G be any d -regular graph and let A note the $n \times n$ adjacency matrix of G . Answer the following questions:
 - (a) (5 points) Show that the largest eigenvalue of A is d .
 - (b) (10 points) Let $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n = d$ denote the eigenvalues of A . What are the eigenvalue of the adjacency matrix corresponding to the complement graph \bar{G} . You may express your answer in terms of $\lambda_1, \dots, \lambda_n, d, n$.
(Complement graph: For every $i \neq j$, edge (i, j) exists in \bar{G} iff edge (i, j) is not present in G)

2. (7 points) Let \mathcal{H} be a hypothesis class consisting of 3-CNF boolean formulas with n variables and k clauses. For a binary classification problem involving n boolean variables, you are told that the target hypothesis belongs to \mathcal{H} . Your goal is to learn a reasonable hypothesis from a randomly sampled example set S . For a given ε and δ , what is the lower bound on S such that with probability at least $(1 - \delta)$, any consistent hypothesis has true error at most ε ?
(*Hint: All you have to do is estimate the size of \mathcal{H} and use the formula that we developed in the class.*)

3. (8 points) Let \mathcal{H} be the hypothesis class consisting of boolean formulas in CNF form over n variables. What is the VC dimension of \mathcal{H} ? Give reasons.

4. (20 points) What is the VC dimension of circles on a plane (\mathbb{R}^2)? Give reasons.
(*You may use the fact that any two circles with different centers intersect in at most two points.*)