COL106: Data Structures and Algorithms

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Greedy Algorithms: Job Scheduling

Problem

Job scheduling: You are given n jobs and you are supposed to schedule these jobs on a machine. Each job i consists of a duration $T(i)$ and a deadline $D(i)$. The *lateness* of a job w.r.t. a schedule is defined as $\max(0, F(i) - D(i))$, where $F(i)$ is the finishing time of job i as per the schedule. The goal is to minimise the maximum lateness.

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  - Smallest jobs first.

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  - Smallest jobs first.
  - Earliest deadline first.

Algorithm

`GreedyJobSchedule`

- Sort the jobs in non-decreasing order of deadlines and schedule the jobs on the machine in this order.

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- Claim 1: There is an optimal schedule with no idle time (time when the machine is idle).

Definition

A schedule is said to have inversion if there are a pair of jobs (i, j) such that

1. $D(i) < D(j)$, and
2. Job j is performed before job i as per the schedule.

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Proof sketch of Claim 2

- Consider an optimal schedule O. First, if there is any idle time, we obtain another optimal schedule O_1 without the idle time.
- Suppose O_1 has inversions. Consider one such inversion (i, j).

- Claim 2.1: If an inversion exists, then there exists a pair of adjacently scheduled jobs (m, n) such that the schedule has an inversion w.r.t. (m, n).

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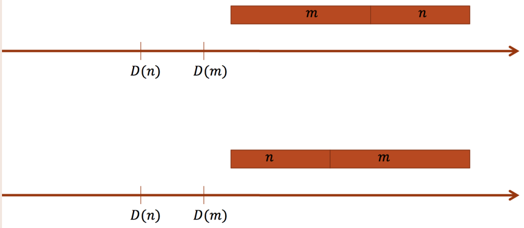
- Claim 2.2: If a schedule has an inversion w.r.t. adjacently scheduled jobs (m, n), then *exchanging* m and n does not increase the maximum lateness.



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