COL106: Data Structures and Algorithms

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Algorithm

- $time \leftarrow 0$

`GraphDFS-with-start-finish(`G`)`
- While there is an "unexplored" vertex u
- `DFS-time(`u`)`

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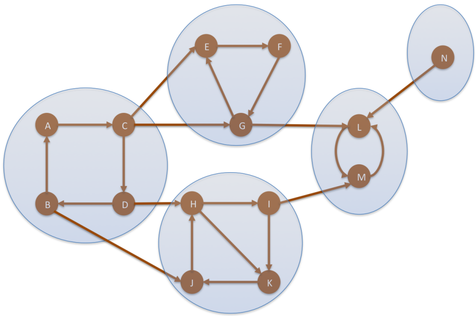
- Mark u as "explored" and set $start(u) \leftarrow ++time$
- While there is an "unexplored" neighbor v of u
  - `DFS-time(`v`)`
- $finish(u) \leftarrow ++time$



Course Overview

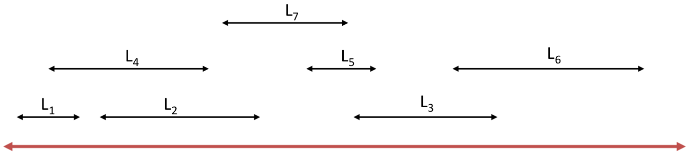
- Material that will be covered in the course:
  - Basic graph algorithms
  - Algorithm Design Techniques
    - Greedy Algorithms
    - Divide and Conquer
    - Dynamic Programming
    - Network Flows
  - Computational intractability

Greedy Algorithms

- *"A local (greedy) decision rule leads to a globally optimal solution."*
- There are two ways to show the above property:
  - Greedy stays ahead
  - Exchange argument

Problem

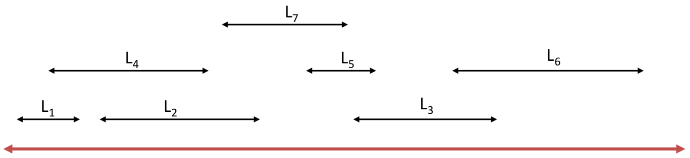
Interval scheduling: Given a set of n intervals of the form $(S(i), F(i))$, find the largest subset of non-overlapping intervals.



Problem

Interval scheduling: Given a set of n intervals of the form $(S(i), F(i))$, find the largest subset of non-overlapping intervals.

- Candidate greedy choices:
  - Earliest start time
  - Smallest duration
  - Least overlapping



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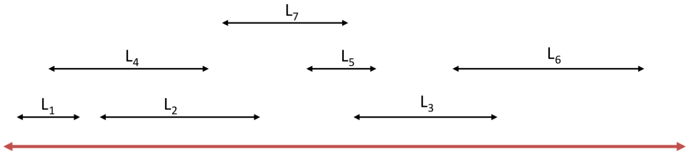
- Candidate greedy choices:
  - Earliest start time
  - Smallest duration
  - Least overlapping
  - Earliest finish time



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Interval scheduling: Given a set of n intervals of the form $(S(i), F(i))$, find the largest subset of non-overlapping intervals.

Algorithm

`GreedySchedule`

- Initialize R to contain all intervals
- While R is not empty
  - Choose an interval $(S(i), F(i))$ from R that has the smallest value of $F(i)$
  - Delete all intervals in R that overlaps with $(S(i), F(i))$

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- Yes we can! We will use "greedy stays ahead" method to show this.

Proof sketch

Let $a_1, a_2, ..., a_k$ be the sequence of requests that `GreedySchedule` picks and $o_1, o_2, ..., o_l$ be the requests in O sorted in non-decreasing order by finishing time.

- Claim 1: $F(a_1) \leq F(o_1)$.

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- Claim 1: $F(a_1) \leq F(o_1)$.
- Claim 2: If $F(a_1) \leq F(o_1)$, $F(a_2) \leq F(o_2)$, ..., $F(a_{i-1}) \leq F(o_{i-1})$, then $F(a_i) \leq F(o_i)$.

- Question: Let O denote some optimal subset and A be the subset given by `GreedySchedule`. Can we show that $A = O$?
- Question Can we show that $|O| = |A|$?
- Yes we can! We will use "greedy stays ahead" method to show this.

Proof sketch

- Let $a_1, a_2, ..., a_k$ be the sequence of requests that `GreedySchedule` picks and $o_1, o_2, ..., o_l$ be the requests in O sorted in non-decreasing order by finishing time.
- We will show by induction that $\forall i, F(a_i) \leq F(o_i)$
  - Claim 1 (base case): $F(a_1) \leq F(o_1)$.
  - Claim 2 (inductive step): If $F(a_1) \leq F(o_1)$, $F(a_2) \leq F(o_2)$, ..., $F(a_{i-1}) \leq F(o_{i-1})$, then $F(a_i) \leq F(o_i)$.
- `GreedySchedule` could not have stopped after a_k.

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- Running time?

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- Running time? $O(n \log n)$

End