COL106: Data Structures and Algorithms

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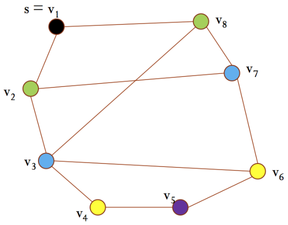
Breadth First Search (BFS)

BFS(G, s)
- $Layer(0) = \{s\}$
- $i \leftarrow 1$
- While(true)
  - Visit all new nodes that have an edge to a vertex in $Layer(i-1)$
  - Put these nodes in the set $Layer(i)$
  - If $Layer(i)$ is empty, then end
  - $i \leftarrow i+1$



- Theorem 1: The shortest path from s to any vertex in $Layer(i)$ is equal to i.

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Proof sketch

- We will prove by induction. Let $P(i)$ denote the statement: *The shortest path from s to any vertex in $Layer(i)$ is equal to i.*
- We will prove that $P(i)$ is true for all i using induction.
- <u>Base case</u>: $P(0)$ is true since $Layer(0)$ contains s.
- <u>Inductive step</u>: Assume $P(0), ..., P(k)$ are true. We will show that $P(k+1)$ is true.
  - Assume for the sake of contradiction that $P(k+1)$ is not true.
  - This implies that there is a vertex v in $Layer(k+1)$ such that the shortest path length from s to v is $< k+1$ (the case $> k+1$ is skipped for class discussion)
  - Consider such a path from s to v. Let u be the vertex in this path just before v.
  - <u>Claim 1</u>: u is contained in $Layer(k)$.
  - This gives us a contradiction since by induction hypothesis, the shortest path length from s to u is k.

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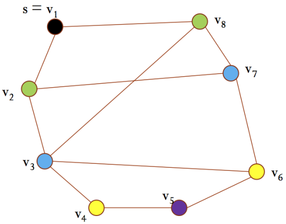

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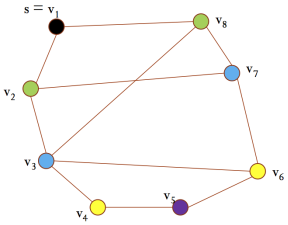
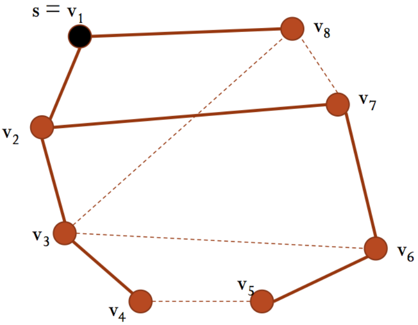


- What is the running time of BFS given that the graph is given in adjacency list representation? $O(n+m)$

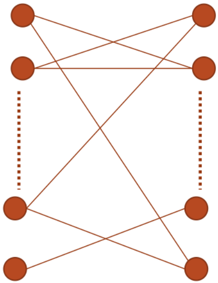
- The BFS algorithm defines the following BFS tree rooted at s
  - Vertex u is the parent of vertex v if u caused the immediate discovery of v.



- Bipartite graph: A graph is *bipartite* iff the vertices can be partitioned into two sets such that there is no edge between any pair of vertices in the same set.

Problem

Given a graph $G = (V, E)$, check if the graph is bipartite.



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Given a graph $G = (V, E)$, check if the graph is bipartite.

- Consider BFS below
- Is it possible that there is an edge between vertices which belong to sets $Layer(i)$ and $Layer(j)$ such that $j - 1 > i$?

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Given a graph $G = (V, E)$, check if the graph is bipartite.

- Is it possible that there is an edge between vertices which belong to sets $Layer(i)$ and $Layer(j)$ such that $j - 1 > i$? No.
- Suppose the given graph contains a cycle of odd length. Can this graph be bipartite?

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Given a graph $G = (V, E)$, check if the graph is bipartite.

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- Suppose the given graph contains a cycle of odd length. Can this graph be bipartite? No.
  - For sake of contradiction assume that the graph is bipartite.
  - Consider a cycle of odd length with nodes numbered $v_1, v_2, ..., v_{2k+1}$.
  - Since the graph is bipartite the nodes may be partitioned into two sets X and Y s.t. there does not exist en edge between nodes in the same partition.
  - If node v_1 is in X, then v_2 has to be in Y, and node v_3 has to be in X and so on. So, node v_{2k+1} has to be in X. But then there is a edge between v_1 and v_{2k+1}.

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- Suppose the given graph contains a cycle of odd length. Can this graph be bipartite? No.
- Can you now use BFS to check if the graph is bipartite?

Problem

Given a graph $G = (V, E)$, check if the graph is bipartite.

- Is it possible that there is an edge between vertices which belong to sets $Layer(i)$ and $Layer(j)$ such that $j - 1 > i$? No.
- Suppose the given graph contains a cycle of odd length. Can this graph be bipartite? No.
- Can you now use BFS to check if the graph is bipartite?

Algorithm

`IsBipartite(G)`

- Run BFS and check if two vertices in the same layer has an edge between them
- If yes then output("no") else output("yes")

Algorithm

`IsBipartite(`G`)`

- Run BFS and check if two vertices in the same layer has an edge between them
- If yes then output("no") else output("yes")

- Claim 1: Any given graph G is bipartite if and only if `IsBipartite(`G`)` outputs "yes".

End