COL106: Data Structures and Algorithms

Ragesh Jaiswal, IIT Delhi

Data Structures: Heaps and Priority Queues

- What is the running time of each of these operations in the array based implementation of Min-Heap?
  - `insert(k, v)`:
  - `min()`:
  - `removeMin()`:

- What is the running time of each of these operations in the array based implementation of Min-Heap?
  - `insert(k, v)`: $O(\log n)$
  - `min()`: $O(1)$
  - `removeMin()`: $O(\log n)$

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- Method 1: Perform n `insert` operations in $O(n \log n)$ time.
- Method 2: Bottom-up heap construction
  - Question: Suppose we have a min-heap H_1 and H_2 both containing $2^h - 1$ entries and an entry E. Can you construct a min-heap for all entries in H_1, H_2 and E combined? What is the running time for your combination algorithm?

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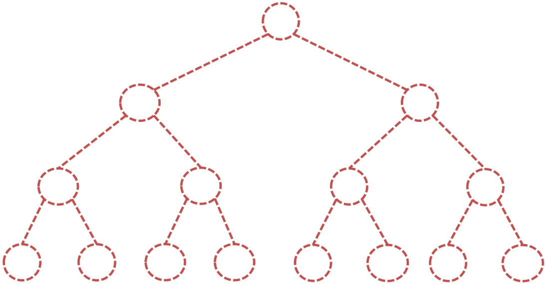

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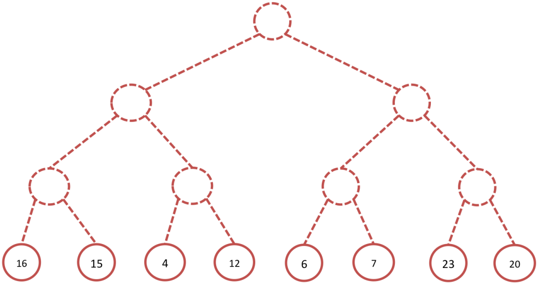

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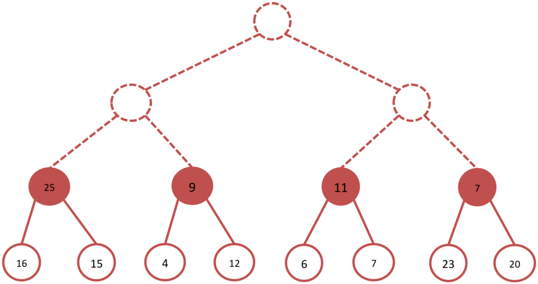

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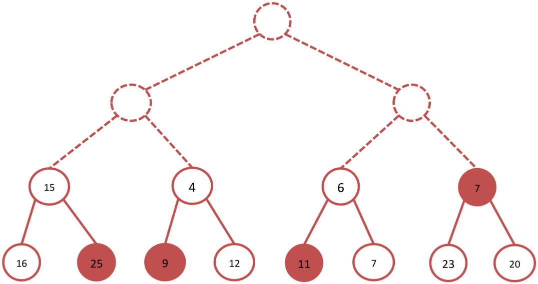

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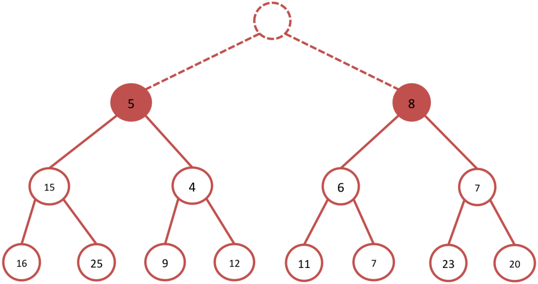

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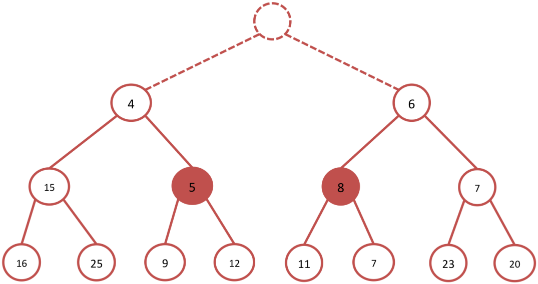

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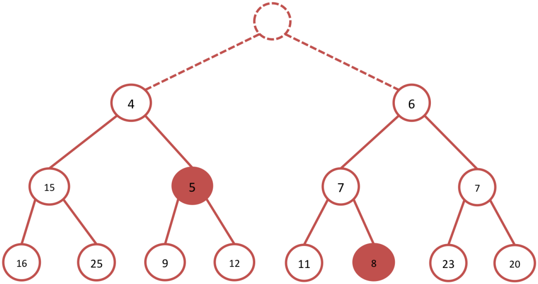

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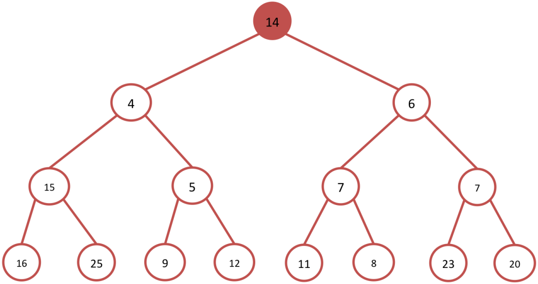

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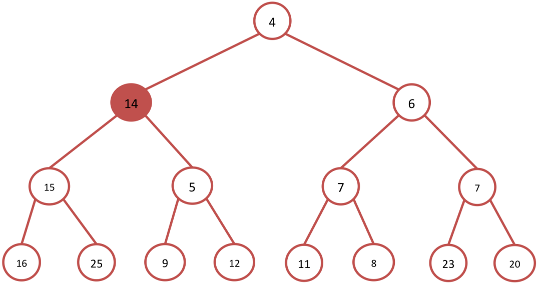

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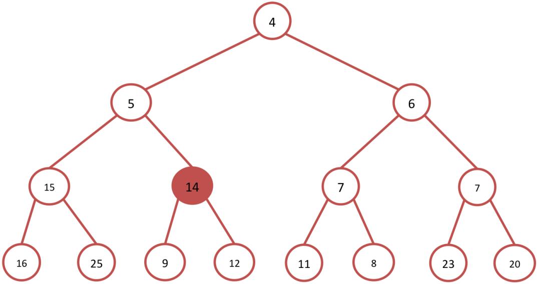

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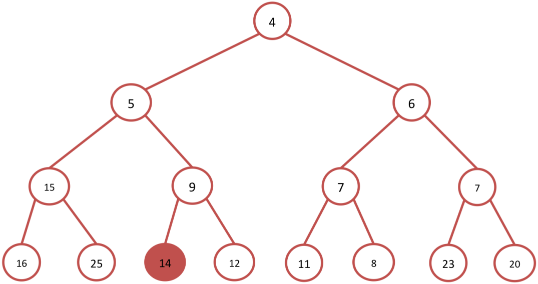

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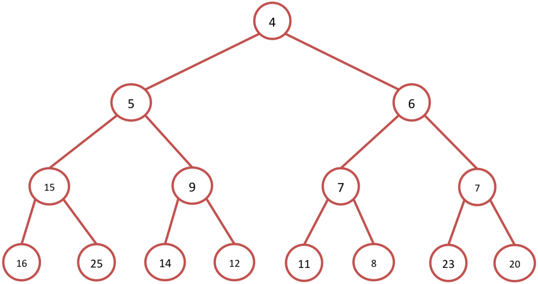

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  - How do we simplify the above expression?

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    - So, the running time of bottom-up heap construction is $O(n)$.

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`HeapSort`(A, n)
- Perform bottom-up heap construction on the array A and let H denote the heap
- for $i = 1$ to n
  - $B[i] \leftarrow H.removeMin()$
- return(B)

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Data Structures: Binary Search Trees

- Suppose we want to store n data entries where each data entry consist of a key (this can be thought of as a unique integer ID) and a value.
- We would like to perform the following operations:
  - get(k): Search an entry with key k and return the value.
  - put(k, v): Associate value v with key k, replacing and returning any existing value in case an entry with key k exists or inserting a new entry if no entry with key k exists.
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- Consider an array based implementation where the elements are NOT sorted based on the keys. What is the running time of:
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- Consider an array based implementation where the elements are sorted based on the keys. What is the running time of:
  - `get(k)`: $O(\log n)$
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- Our next goal is to build a data structure where the running time of the operations are:
  - get(k): $O(\log n)$
  - put(k, v): $O(\log n)$
  - remove(k): $O(\log n)$

- Binary Search Tree: Binary Search Trees are proper binary trees such that each internal node p stores a key-value pair such that:
  1. Keys stored in the left sub-tree of p are less than k
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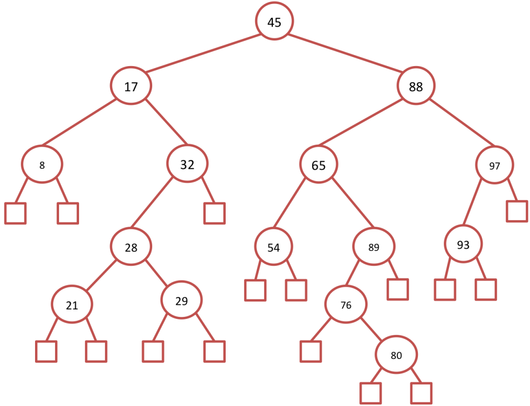
- Is the tree below a binary search tree?



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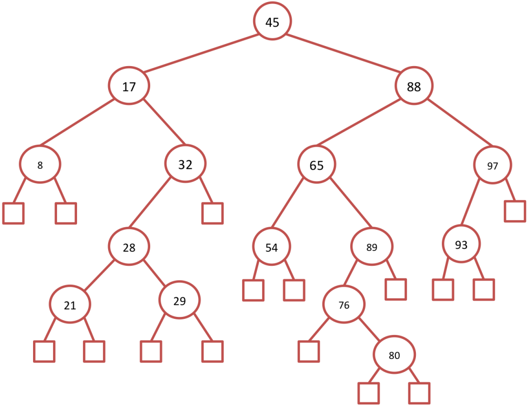

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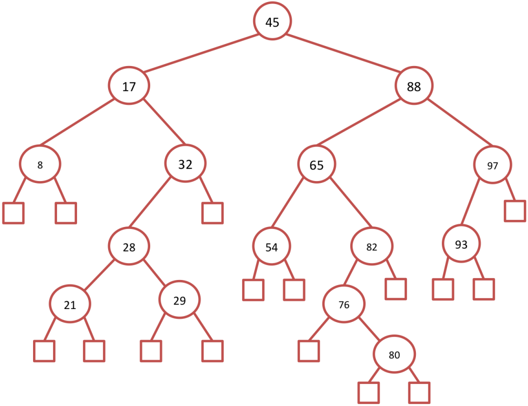

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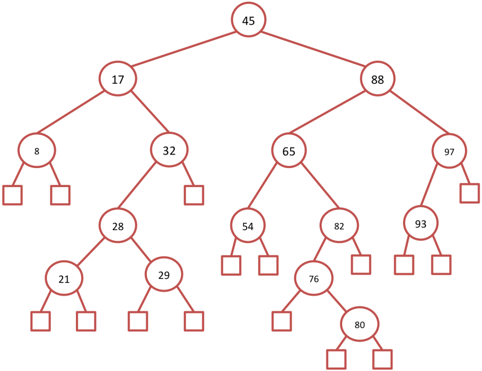

Figure : The "value" of entries are not shown.

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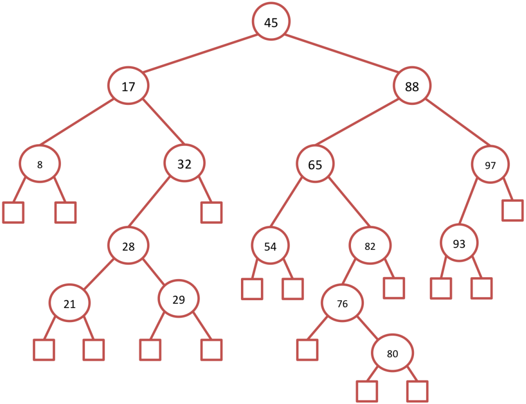

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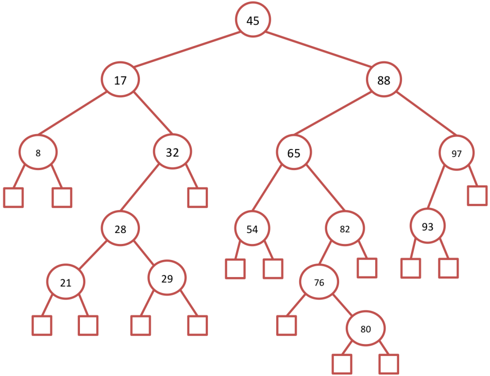
- Question: Given a binary search tree, how do we perform `get(k)`?
  - How do we search for the key 68 in the binary search tree below?



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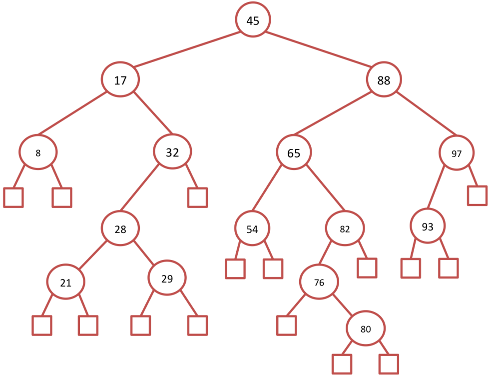
  - Now try searching 76 in the tree.



- Consider the following implementation:

Code

```
class Node{
   public int key;
   public String value;
   public Node leftChild;
   public Node rightChild;
   public Node parent;
}
public class BST{
   public int size;
   public Node root;
   public BST(){
       size = 0;root = null;
   }
   public boolean isLeaf(Node N){//To be written}
   public String get(int k){//To be written}
}
```

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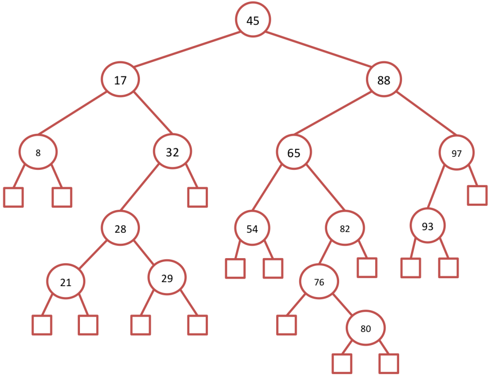
- Question: Given a binary search tree, how do we perform `put(k, v)`?
  - Suppose we want to perform `put(68, "A")` in the BST below.



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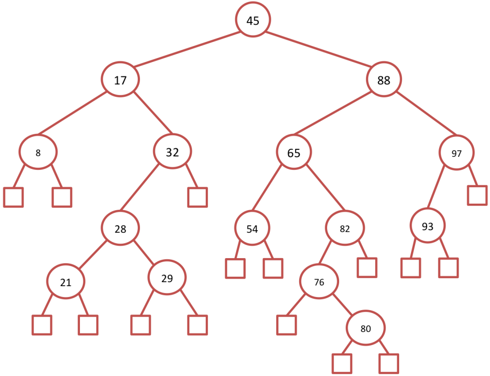

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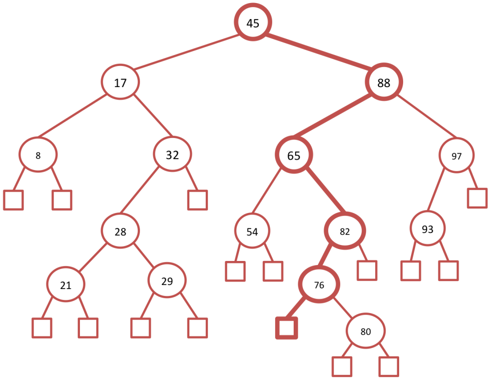

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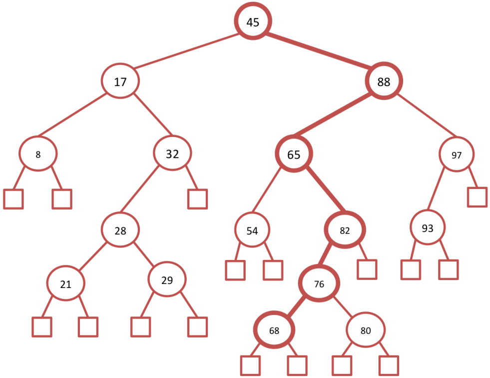

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public class BST{
  public int size;
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    size = 0;root = null;
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- Binary Search Tree: Binary Search Trees are proper binary trees such that each internal node p stores a key-value pair such that:
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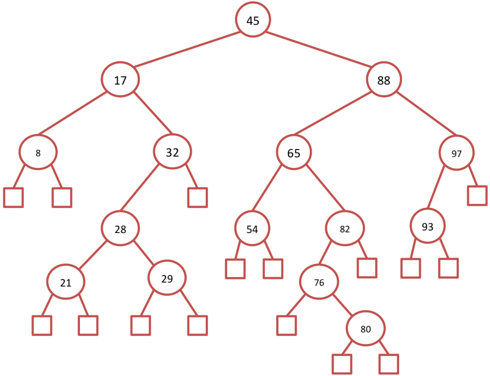
  - Suppose we want to perform `remove(32)` in the BST below.



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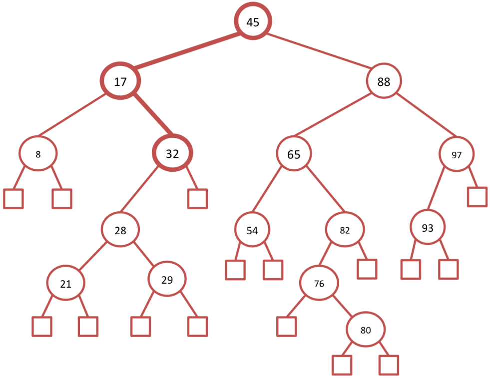

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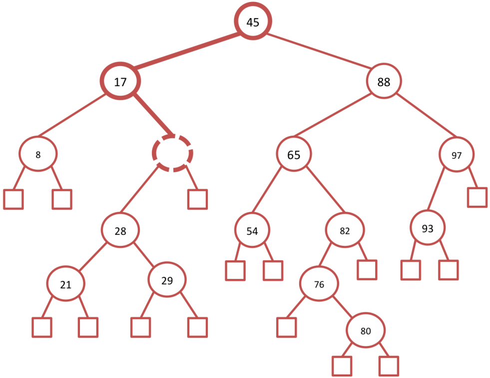

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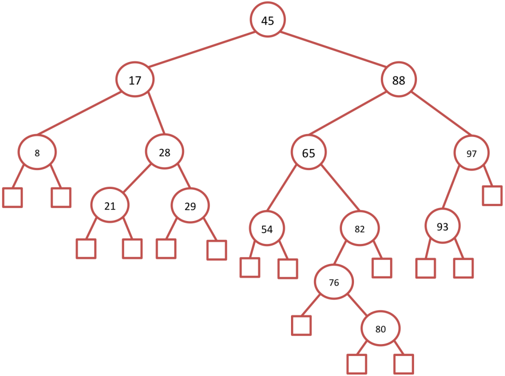

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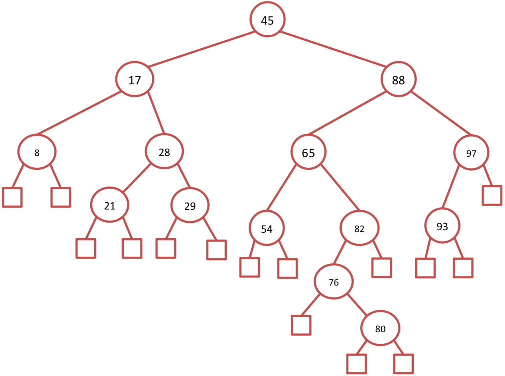

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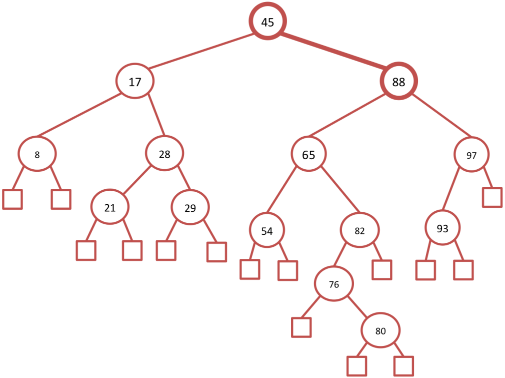

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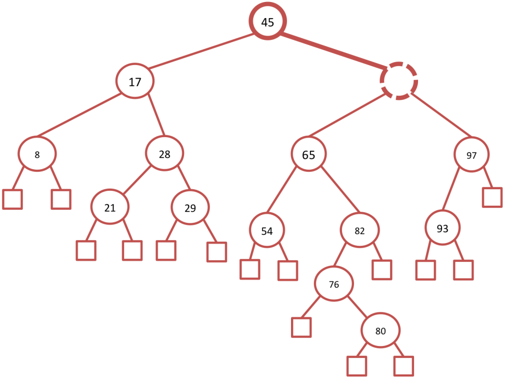

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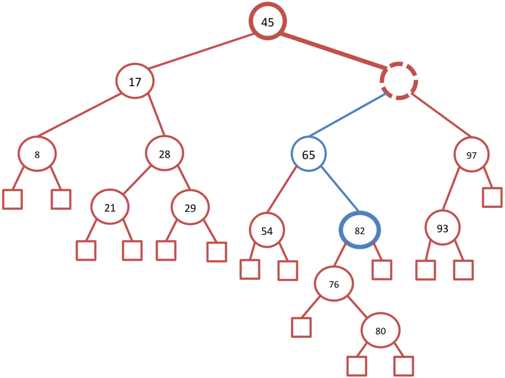

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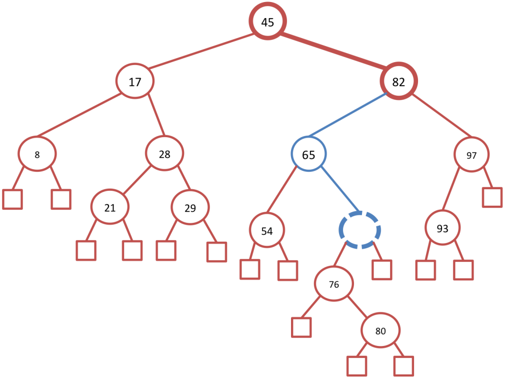

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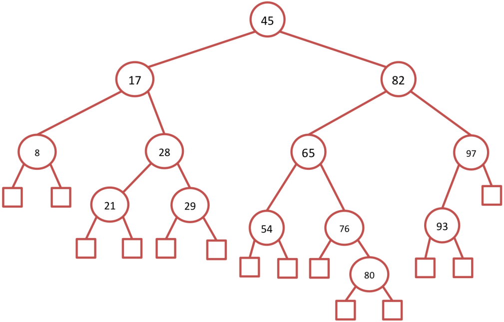

- Consider the following implementation:

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- What is the worst case running time of each of the following operations when the BST is balanced?
  - `get(k)`:
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- A BST is perfectly balanced if for every internal node, there are equal number of nodes in its left and right sub-trees.

- What is the worst case running time of each of the following operations when the BST is balanced?
  - `get(k)`: $O(\log n)$
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- So, our next goal shall be to build balanced binary search trees.

Data Structures: Balanced Binary Search Trees

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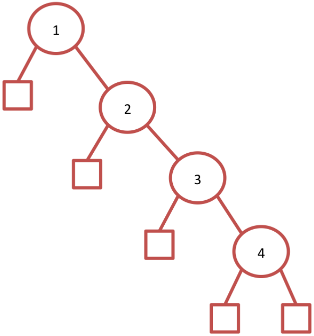
- Suppose we start with an empty BST and insert the keys $1, 2, 3, 4$, then the BST obtained is shown below.



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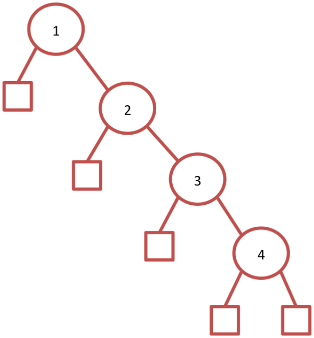
- This tree is not balanced. Can you think of a way to balance this tree?



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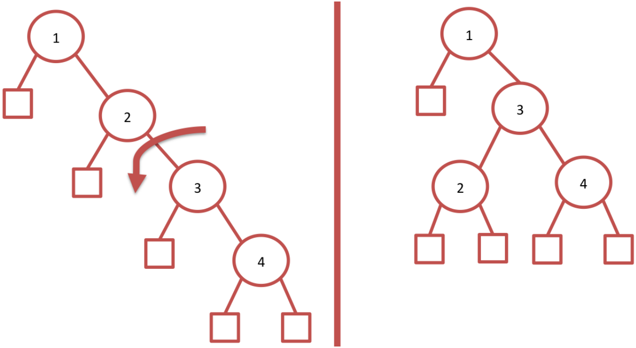
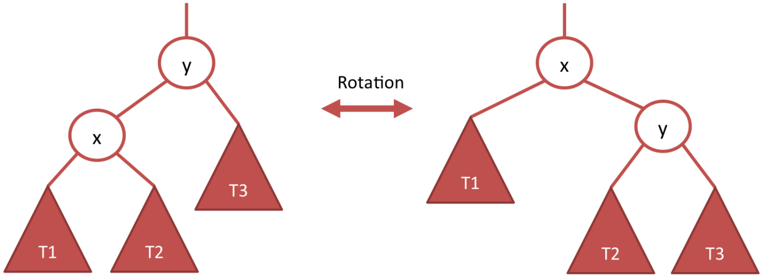


- Rotation for tree balancing.



- Tri-node restructuring for a node x, its parent y, and its grandparent z.

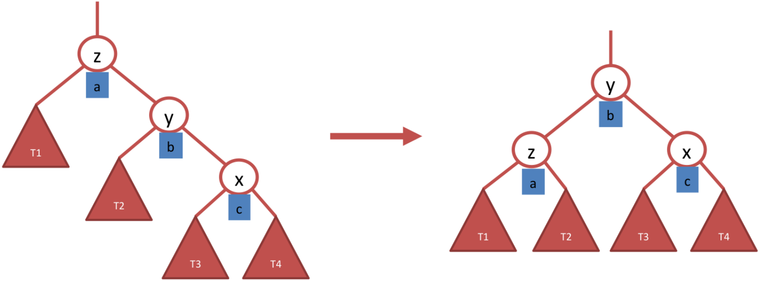


Figure : Case #1

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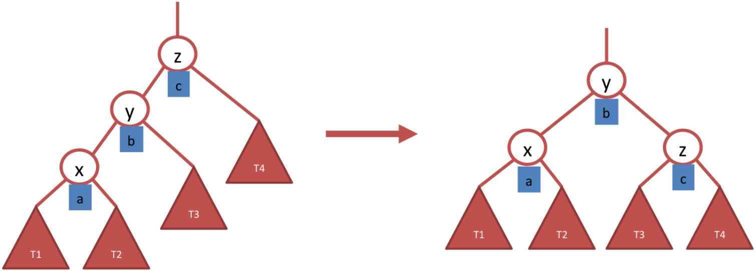

Figure : Case #2

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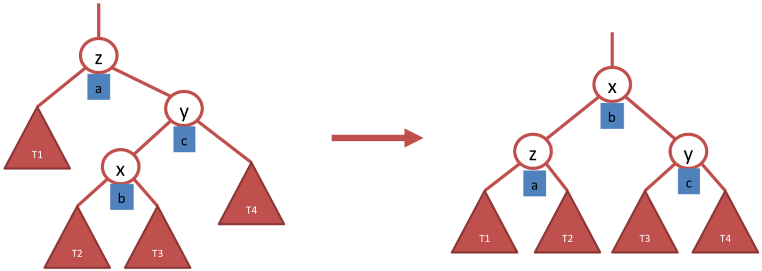

Figure : Case #3

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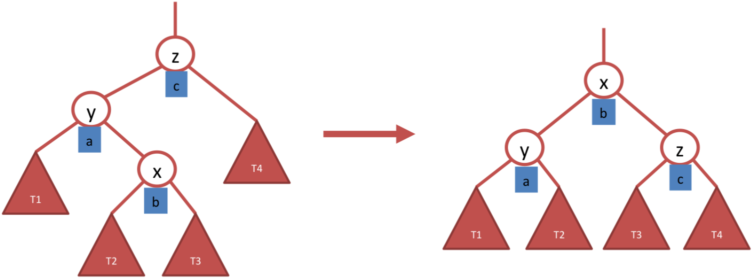

Figure : Case #4

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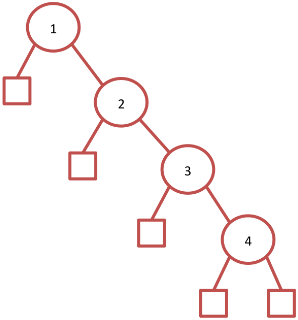

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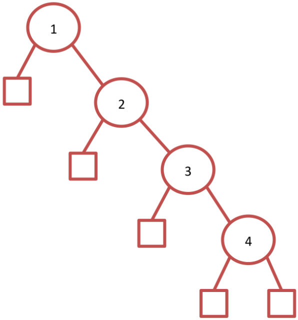

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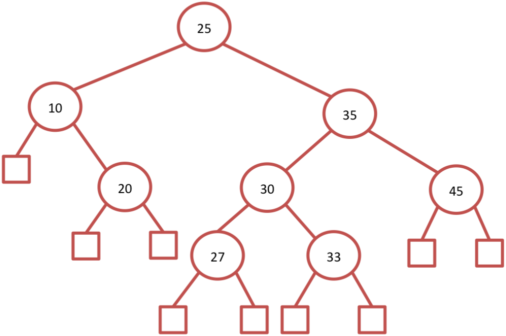

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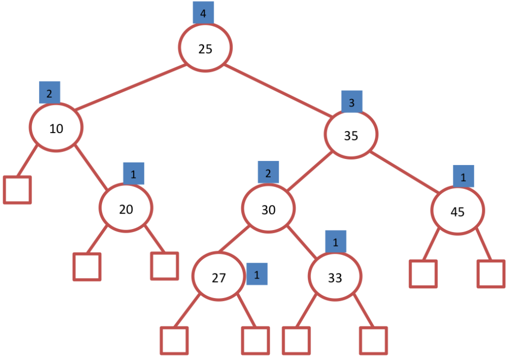

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- Claim: The height of any AVL tree storing n nodes is $O(\log n)$.

End