COL106: Data Structures and Algorithms

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Data Structures: Heaps and Priority Queues

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  - `min()`: $O(1)$
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- Note that a Max-Heap can be defined in a similar manner as a Min-Heap.

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  - Array elements are typically stored in contiguous locations in the memory and this has its advantages.

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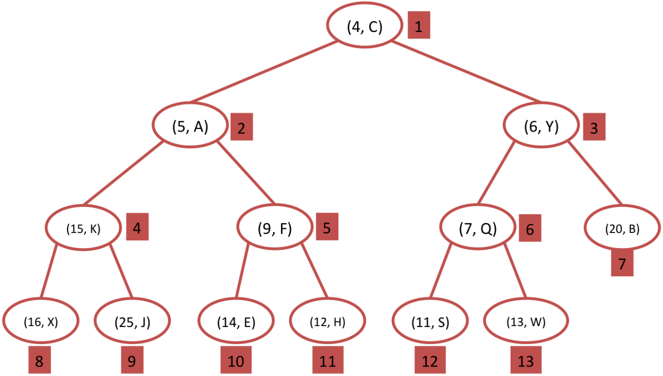
- Do you observe some pattern in the labels?



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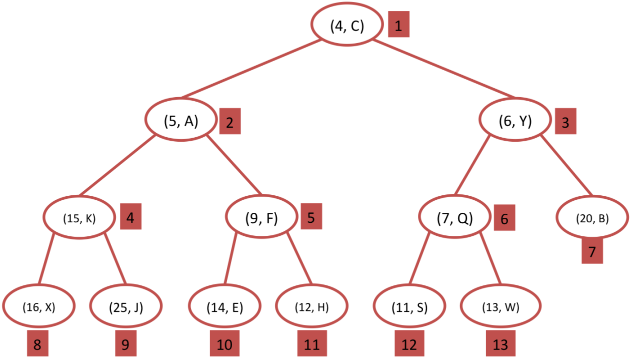

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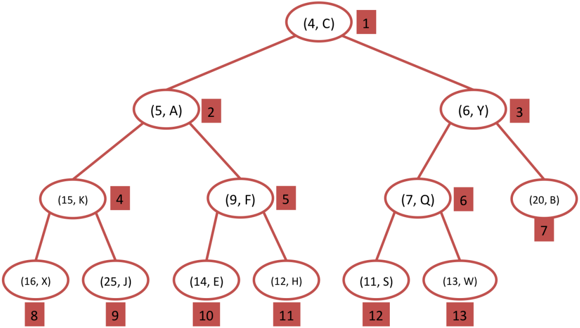

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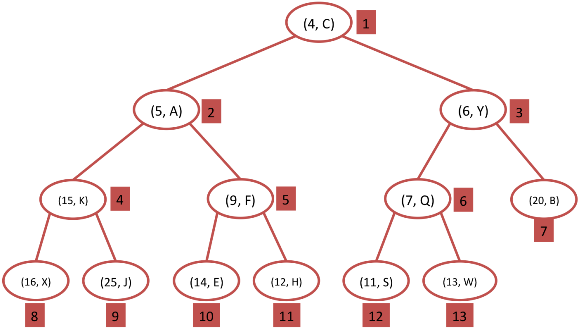

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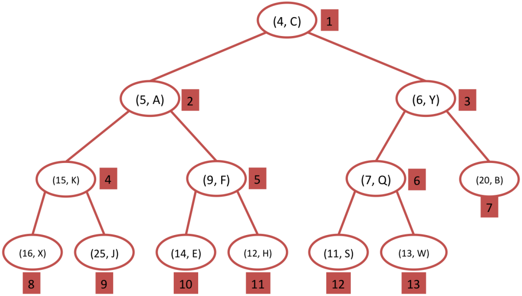
- Main idea: Store node labeled i at index i of the array.



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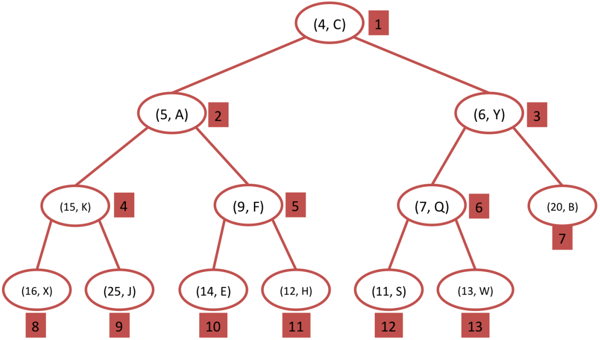

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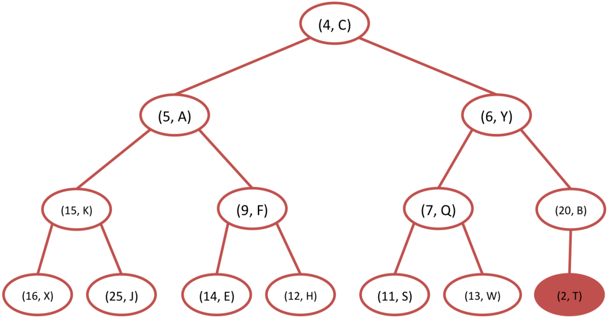

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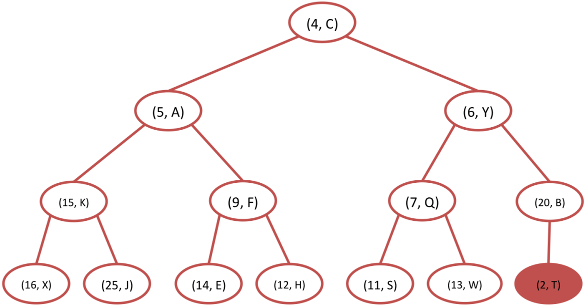

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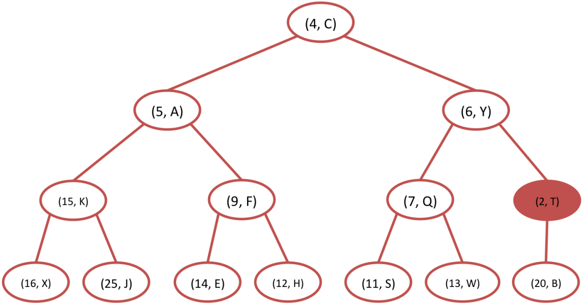

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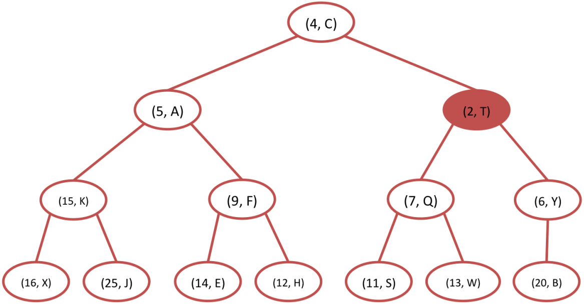

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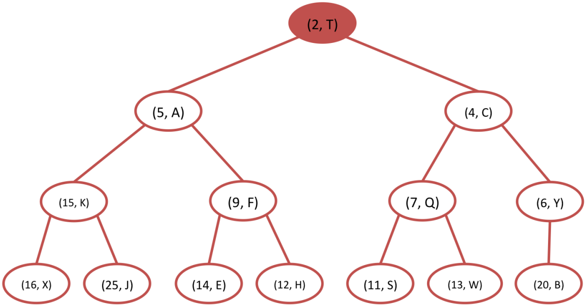

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- Consider an array based implementation of min-heap.

Implementation

```
class Entry{
  public int key;
  public String value;
  public Entry(int k, String v){
     key = k;
     value = v;
  }
}
public class MinHeapArray{
  final int MAX_HEAP_SIZE = 1000;
  public Entry[] A;
  public int size;
  public MinHeapArray(){
     size = 0;
     A = new Entry[MAX_HEAP_SIZE];
  }
  public void upHeapBubble(int i){//To be written}
  public void downHeapBubble(int i){//To be written}
  public String min(){//To be written}
  public void insert(int k, String v){//To be written}
  public String removeMin(){//To be written}
}
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Given n entries create a min-heap of these entries.

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- Method 1: Perform n `insert` operations in $O(n \log n)$ time.
- Method 2: Bottom-up heap construction
  - Question: Suppose we have a min-heap H_1 and H_2 both containing $2^h - 1$ entries and an entry E. Can you construct a min-heap for all entries in H_1, H_2 and E combined? What is the running time for your combination algorithm?

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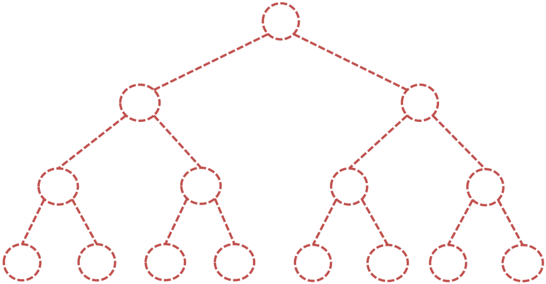

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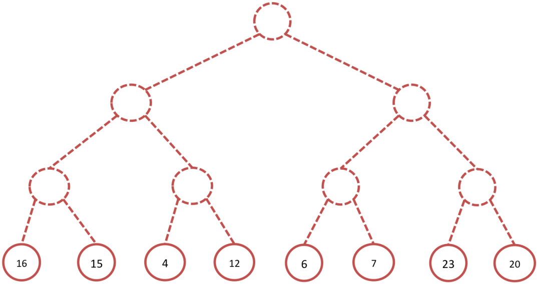

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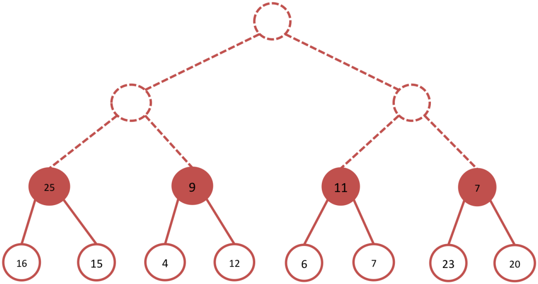

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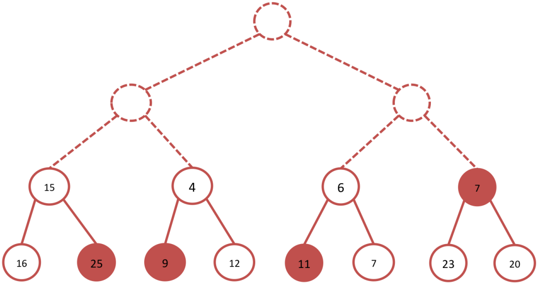

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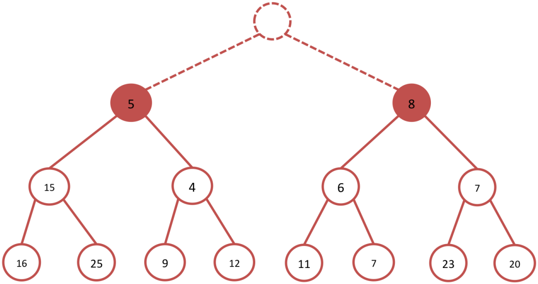

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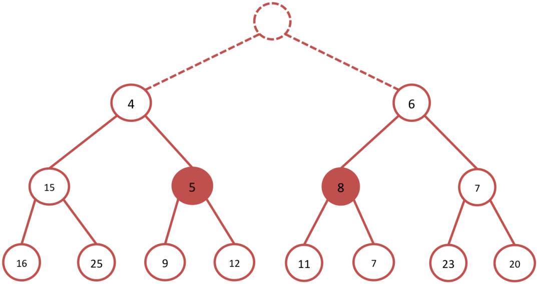

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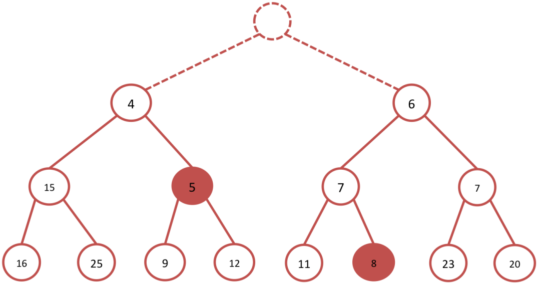

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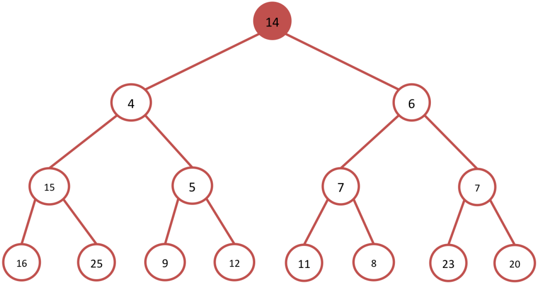

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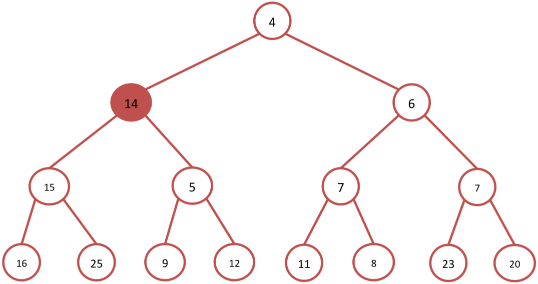

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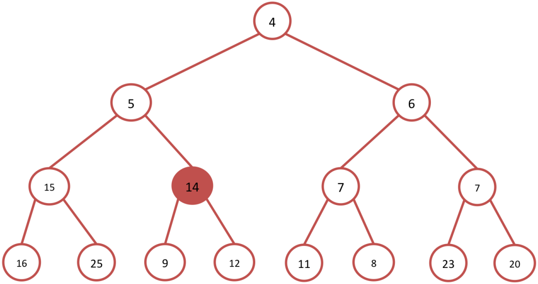

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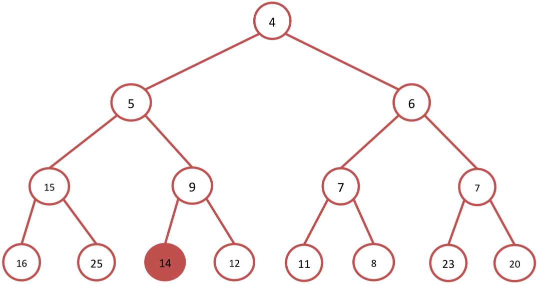

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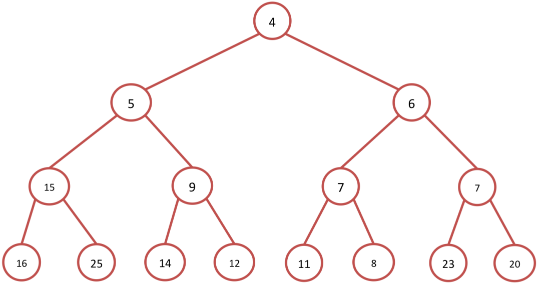

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  - Suppose this construction is performed on an array with $n = 2^{h+1} - 1$ entries. What is the running time?

End