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**COL351: Analysis and Design of Algorithms****Instructor:** Ragesh Jaiswal

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1. Given a string  $S = "s_1s_2\dots s_n"$ , design an algorithm to find the minimum number of characters that need to be inserted to make the resulting string a palindrome. Consider the example  $S = "abcd"$ . We can get palindrome **"adbcbda"** or palindrome **"dabcbad"** by inserting two characters (one **a** and one **d**). Moreover, we cannot get a palindrome by inserting just one character. Discuss running time of your algorithm.
2. You are given an  $n \times 5$  matrix  $A$  consisting of integers (positive or negative). You have to design an algorithm that outputs a set  $S$  of tuples  $(i, j)$  indicating locations of the 2-D matrix  $A$  such that:
  - (a)  $\sum_{(i,j) \in S} A[i, j]$  is maximized, and
  - (b) For all pairs of tuples  $(i_1, j_1), (i_2, j_2) \in S$ ,  $(i_2, j_2) \notin \{(i_1 - 1, j_1), (i_1 + 1, j_1), (i_1, j_1 - 1), (i_1, j_1 + 1)\}$ .

Discuss running time and correctness proof.

3. You are given an array  $A = A[1], A[2], \dots, A[n]$  containing  $n$  integers and a positive integer  $k$ . Design an algorithm that outputs an array  $C = C[1], C[2], \dots, C[k]$  of size  $k$  such that

$$\sum_{i=1}^n \min_{j \in \{1, \dots, k\}} \{|A[i] - C[j]|\}$$

is minimized. Here  $|x - y|$  denotes the absolute value of the difference between  $x$  and  $y$ . Discuss running time for your algorithm.

(For example, if  $A = [0, 1, 2, 10, 11, 12]$  and  $k = 2$ , then  $C = [1, 11]$ )