

There are 1 questions for a total of 25 points.

- (25) 1. Binary Multiplication: Given two n -bit integers a and b , you have to implement two algorithms:
- Long Multiplication: Implement the brute-force $O(n^2)$ algorithm that simulates the long multiplication.
 - Karatsuba: Implement the $O(n^{\log_2 3})$ -time algorithm discussed in class to multiply a and b .

Your programs should take input from a file named `input.txt` and should write the output in a file named `output.txt`. Your programs should produce an output within 15 seconds for this assignment. The format for input and output files are as follows.

INPUT: The first line of the input file gives the value of n (assume $n \leq 10000$). The next line gives the value of a . The final line gives the value of b . Below is an example of an input file.

```
3
111
111
```

OUTPUT: The output should be $a \times b$. For example, consider the output file corresponding to the input file above:

```
110001
```

GRAPH PLOTTING: For $n = 1, 2, 2^2, 2^3, 2^4, 2^5, \dots, 2^{16}$, record the running time of your algorithms for all 1's input. That is, record time for $1 \times 1, 11 \times 11, 1111 \times 1111$ and so on. Plot running time versus n for both the algorithms on the same graph (running time is on the Y -axis and n on the X -axis).

SUBMISSION INSTRUCTIONS: All your program files should be in a directory named `<Entry no.>`. You will be asked to create a zip of this directory and submit this zip file. In this directory, there should be a makefile that will compile your code (read about makefile on the net in case you do not know what it is). After running `make`, the directory should have executables called `longmultiplication` and `karatsuba`. This when executed, should read the input file (`input.txt`) and write the answer in the output file (`output.txt`).