Name: _____

Entry number: _____

- You may use any of the following known NP-complete problems to show that a given problem is NP-complete: 3-SAT, INDEPENDENT-SET, VERTEX-COVER, SET-COVER, HAMILTONIAN-CYCLE, HAMILTONIAN-PATH, SUBSET-SUM, 3-COLORING, CLIQUE.

There are 5 questions for a total of 100 points.

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1. (20 points) Consider the following problem:

   F-SAT: Given a boolean formula in CNF form such that (i) each clause has exactly 3 terms and (ii) each variable appears in at most 3 clauses (including in negated form), determine if the formula is satisfiable.

   Answer the following questions with respect to the above problem under the assumptions (i) $\mathsf{P} = \mathsf{NP}$, and (ii) $\mathsf{P} \neq \mathsf{NP}$. Give reasons.

   (a) Is F-SAT $\in$ NP?

   (b) Is F-SAT NP-complete?

   (c) Is F-SAT NP-hard?

   (d) Is F-SAT $\in$ P?

2. (15 points) Consider the following problem:

   > LONG-PATH: Given a weighted, directed graph $G = (V, E)$, two vertices $s, t \in V$ and a number W, determine if there is a simple path between s and t such that the sum of weights of edges in this path is $\geq W$.

   Recall that a simple path is a path that does not have any vertices repeated. Show that LONG-PATH is NP-Complete.

3. (15 points) Consider the following problem:

   CUT: Given a rectangular piece of cloth sheet of dimension $h \times w$, n rectangular items with dimension $h_1 \times w_1, ..., h_n \times w_n$ and profits $p_1, ..., p_n$, and an integer P, determine whether it is possible to cut these rectangular items from the sheet such that the total profit is exactly P. You are allowed to cut at most one copy of any item from the sheet. You may assume that arbitrary cuts are possible.

   Show that CUT is NP-complete.

4. (25 points) Consider the following problem:

   SPAN-TREE: Given an undirected graph $G = (V, E)$, determine if there is a spanning tree of G that has at most 10 leaves.

   Either show that this problem is NP-complete or give a polynomial time algorithm (along with correctness proof and running time).

5. (25 points) A cycle cover of a given directed graph $G = (V, E)$ is a set of vertex disjoint cycles that cover all the vertices. Consider the following problem:

   CYCLE-COVER: Given a directed graph $G = (V, E)$, determine if there is a cycle cover of G.

   Either show that this problem is NP-complete or give a polynomial time algorithm (along with correctness proof and running time).