Name: ______________________________

Entry number: ________________________

There are 5 questions for a total of 75 points.

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1. You are given n points $X \subseteq \mathbb{R}^d$ (i.e., n points in d-dimensional Euclidean space) and a positive integer k. You are asked to design an algorithm to find a subset $C \subseteq X$ of size k such that the following quantity is minimized:

$$\max_{x \in X} \{\min_{c \in C} \{||x - c||\}\}$$

(*That is, a subset C of X containing k points such that the maximum distance of a point in X to the nearest point in the set C is minimized.*)

Consider the following greedy algorithm for this problem.

`GreedySelect`(X, k)
- Let c_1 be an arbitrary point in X
- $C \leftarrow \{c_1\}$
- for $i = 2$ to k
  - Pick a point $c_i \in X$ that is farthest from points in C.
    (*That is,* $c_i = \arg\max_{x \in X} \{\min_{c \in C} \{||x - c||\}\}$)
  - $C \leftarrow C \cup \{c_i\}$
- return(C)

(a) (5 points) Show that the above greedy algorithm does not always produce an optimal solution.

(b) (10 points) Show that `GreedySelect` gives an approximation guarantee of 2.

2. (15 points) You are given n points on a two-dimensional plane. A point (x, y) is said to *dominate* (x', y') iff $x > x'$ and $y > y'$. Design an algorithm to output all points that are not dominated by any other point. Give pseudocode, discuss running time, and give proof of correctness.

3. (15 points) Let S and T be sorted arrays each containing n elements. Give an algorithm to find the n^{th} smallest element out of the $2n$ elements in S and T . Give pseudocode, discuss running time, and give proof of correctness.

4. (20 points) This is problem no. 5 from Chapter 5 in the book. You are given n non-vertical lines in the plane, labeled $L_1, ..., L_n$, with the i^{th} line specified by the equation $y = a_i x + b_i$. We will make the assumption that no three of the lines meet at a single point. We say line L_i is *uppermost* at a given x-coordinate x_0 if its y-coordinate at x_0 is greater than the y-coordinates of all the other lines at x_0: $a_i x_0 + b_i > a_j x_0 + b_j$ for all $j \neq i$. We say line L_i is *visible* if there is some x-coordinate at which it is uppermost – intuitively some portion of it can be seen if you look down from "$y = \infty$". Design an algorithm that takes n lines as input and returns all the lines that are visible. Give pseudocode, discuss running time, and give proof of correctness.

5. (10 points) You are given an unsorted integer array A containing n distinct integers (assume n is a power of 2). You are supposed to design an algorithm that finds the minimum and the second minimum element in the array A. The running time for this problem is measured in terms of the number of pairwise comparisons that your algorithm performs in the worst case. (*Note that you can find the minimum and the second minimum using* $(2n-3)$ *comparisons by a linear scan.*) Design an algorithm that solves the problem using at most $(n + \log n - 2)$ comparisons. Give pseudocode. Discuss correctness and argue why the number of comparisons is at most $(n + \log n - 2)$.