Name: ______________________

Entry number: ______________________

There are 4 questions for a total of 75 points.

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1. (20 points) Given a weighted, undirected graph G and a minimum spanning tree T of G. Suppose that we decrease the weight of one of the edges not in T. Design an algorithm for finding the minimum spanning tree in the modified graph. Give pseudocode, discuss running time, and give proof of correctness.

2. (15 points) Let T be a minimum spanning tree of a weighted, undirected graph G. Given a connected subgraph H of G, show that $T \cap H$ is contained in some minimum spanning tree of H.

3. There is a currency system that has coins of value $v_1, v_2, ..., v_k$ for some integer $k > 1$ such that $v_1 = 1$. You have to pay a person V units of money using this currency. Answer the following:
   (a) (16 points) Let $v_2 = c^1, v_3 = c^2, ..., v_k = c^{k-1}$ for some fixed integer constant $c > 1$. Design a greedy algorithm that minimises the total number of coins needed to pay V units of money for any given V. Give pseudocode, discuss running time, and give proof of correctness.

(b) (4 points) Let $c > 1$ be any fixed integer constant. Does your greedy algorithm above also work when for all $1 \leq i < k$, $\frac{v_{i+1}}{v_i} \geq c$? Give reason for your answer.

4. (20 points) Given a list of n natural numbers $d_1, d_2, ..., d_n$, design an algorithm that determines whether there exists an undirected graph $G = (V, E)$ whose vertex degrees are precisely $d_1, ..., d_n$. That is, if $V = \{v_1, ..., v_n\}$, then degree of v_i should be exactly d_i. G should not contain multiple edges between the same pair of nodes or "loop" edges. Give pseudocode, discuss running time, and give proof of correctness.