Consider the following 3-partition problem. Given integers $a_1, a_2, \dots a_n$, we want to determine whether it is possible to partition $\{1,2,\dots,n\}$ into three disjoint subsets I,J,K such that

$$\sum_{i\in I} a_i = \sum_{j\in J} a_j = \sum_{k\in K} a_k = \frac{1}{3}\sum_{i=1}^{n} a_i$$

For example, for input (1,2,3,4,4,5,8) the answer is *yes* because there is the partition (1,8), (4,5), (2,3,4). On the other hand, for input (2,2,3,5) the answer is *no*. Give an efficient algorithm for this problem. (10)

We solve this by Dynamic Programming. Let X be a 3-dimensional array of size $n \times W \times W$ where $W = \frac{1}{3}\sum_{i=1}^{n} a_i$.

Define $X[i,j,k] = 1$ if $\exists\ S_1, S_2 \subseteq \{a_1, a_2 \dots a_i\}$, $S_1 \cap S_2 = \phi$ and $\sum_{a_i \in S_1} a_i = j$ & $\sum_{a_i \in S_2} a_i = k$.

$= 0$ otherwise.

Now,

$X[i,j,k] = 1$ if $X[i-1,j,k] = 1$ OR $X[i-1, j-a_i, k] = 1$ OR $X[i-1, j, k-a_i] = 1$.

Hence the entire array can be filled in time $O(nW^2)$.

Finally, the answer to the 3-partition problem is YES if $X[n, \frac{W}{3}, \frac{W}{3}] = 1$

COMMON MISTAKES: Many students have given the following solution:

Find a subset S whose sum is W, remove these integers & find another subset of sum W.

For the instance (1,2,3,4,4,5,8) if the first subset found is (1,3,5) Then the remaining integers are (2,4,4,8) & there is no subset here of sum 9. However the original instance is a YES instance as shown above (in the question).

There is no way of formulating this as a max flow problem. Some students have created networks with edge capacity equal to the given integers. There is no way to ensure that these edges would be saturated.

MARKING SCHEME:

2 marks given if you gave the "wrong solution" described above i.e. find a set S of sum W, remove it & repeat.