Tutorial Sheet 9

Oct 5,6,7

1. Find a recurrence relation for the number of ternary strings of length n that contain either two consecutive 0s or two consecutive 1s.

2. Find a recurrence relation for the number of bit strings of length n that contain the string 01.

3. Find the recurrence relation satisfied by R_n, where R_n is the number of regions that a plane is divided into by n lines, if no two of the lines are parallel and no three of the lines go through the same point.

4. In the Tower of Hanoi puzzle, suppose our goal is to transfer all n disks from peg 1 to peg 3, but we cannot move a disk directly between pegs 1 and 3. Each move of a disk must be a move involving peg 2. As usual, we cannot place a disk on top of a smaller disk. Find a recurrence relation for the number of moves required to solve the puzzle for n disks with this added restriction.

5. Let A_n be the $n \times n$ matrix with 2's on its main diagonal, 1's in all positions next to a diagonal element, and 0's everywhere else. Find a recurrence relation for d_n, the determinant of A_n. Solve this recurrence relation to find a formula for d_n.

6. Let $S(m,n)$ denote the number of onto functions from a set with m elements to a set with n elements. Show that$S(m,n)$ satisfies the recurrence relation $S(m,n) = n^m - \sum_{k=1}^{n-1} C(n,k)S(m,k)$ whenever $m \geq n$ and $n > 1$, with the initial condition $S(m,1) = 1$.