

# Tutorial Sheet 8

Sept 28, 30, Oct 1

1. How many  $n$ -digit decimal sequences (using the digits 0 – 9) are there in which the digits 1, 2 and 3 all appear?
2. How many arrangements of 52 letters, 2 As, 2 Bs, 2 Cs, etc. are there with no pair of consecutive letters the same?
3. How many ways are there of dealing a 13 card hand with at least one void in a suit?
4. The Mobius function  $\mu(n)$  is 0 if  $n$  contains a square factor and is  $(-1)^r$  if  $n$  is the product of  $r$  different primes. For any  $n \geq 2$ , prove that  $\sum_{d|n} \mu(d) = 0$ .
5. If 8 dies are rolled what is the probability that all 6 numbers appear?
6. How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s.
7. Use the product rule to show that there are  $2^{2^n}$  different truth tables for propositions in  $n$  variables.
8. Find a formula for the number of ways to seat  $r$  of  $n$  people around a circular table, where seatings are considered the same if every person has the same two neighbors without regard to which side these neighbors are sitting on.
9. Let  $n$  and  $k$  be integers with  $1 \leq k \leq n$ . Show that 
$$\sum_{k=1}^n \binom{n}{k} \binom{n}{k-1} = \binom{2n+2}{n+1} / 2 - \binom{2n}{n}$$
10. How many ways are there to distribute six indistinguishable objects into four indistinguishable boxes so that each of the boxes contains at least one object?
11. Suppose that  $S$  is a set with  $n$  elements. How many ordered pairs  $(A, B)$  are there such that  $A$  and  $B$  are subsets of  $S$  with  $A \subseteq B$ ? [Hint: Show that each element of  $S$  belongs to  $A, B - A$ , or  $S - B$ .]