

Tutorial Sheet 5

August 24, 26 and 27

1. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.
 - (a) How many socks must he take out to be sure that he has at least two socks of the same color?
 - (b) How many socks must he take out to be sure that he has at least two black socks?
2. [*] Let $(x_i, y_i), i = 1, 2, 3, 4, 5$ be a set of five distinct points with integer coordinates in the xy plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates.
3. How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \bmod 5 = a_2 \bmod 5$ and $b_1 \bmod 5 = b_2 \bmod 5$?
4. [*] Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.
5. [**] Suppose that 21 girls and 21 boys enter a mathematics competition. Furthermore, suppose that each entrant solves at most six questions, and for every boy-girl pair, there is at least one question that they both solved. Show that there is a question that was solved by at least three girls and at least three boys.
6. Show that in a group of 10 people (where any two people are either friends or enemies), there are either three mutual friends or four mutual enemies, and there are either three mutual enemies or four mutual friends. Use this to show that among any group of 20 people (where any two people are either friends or enemies), there are either four mutual friends or four mutual enemies.
7. Find the least number of cables required to connect eight computers to four printers to guarantee that for every choice of four of the eight computers, these four computers can directly access four different printers. Justify your answer.
8. [*] An arm wrestler is the champion for a period of 75 hours. (Here, by an hour, we mean a period starting from an exact hour, such as 1 p.m., until the next hour.) The arm wrestler had at least one match an hour, but no more than 125 total matches. Show that there is a period of consecutive hours during which the arm wrestler had exactly 24 matches. Is the statement true if 24 is replaced by
 - (a) 2?
 - (b) 23?
 - (c) 25?
 - (d) 30?
9. [*] Let x be an irrational number. Show that for some positive integer j not exceeding the positive integer n , the absolute value of the difference between jx and the nearest integer to jx is less than $1/n$.