Tutorial Sheet 3

Aug 10, 12, 13

1. Let $a_1, a_2, ..., a_n$ be positive real numbers. The arithmetic mean of these numbers is defined by $A = (a_1 + a_2 + \cdots + a_n)/n$, and the geometric mean of these numbers is defined by $G = (a_1 a_2 \ldots a_n)^{1/n}$. Use mathematical induction to prove that $A \geq G$.

2. Show that it is possible to arrange the numbers $1, 2, ..., n$ in a row so that the average of any two of these numbers never appears between them. [Hint: Show that it suffices to prove this fact when n is a power of 2. Then use mathematical induction to prove the result when n is a power of 2.]

3. Suppose that we want to prove that $\frac{1}{2} \cdot \frac{3}{4} \cdots \frac{2n-1}{2n} < \frac{1}{\sqrt{3n}}$ for all positive integers n.

   (a) Show that if we try to prove this inequality using mathematical induction, the basis step works, but the inductive step fails.

   (b) Show that mathematical induction can be used to prove the stronger inequality $\frac{1}{2} \cdot \frac{3}{4} \cdots \frac{2n-1}{2n} < \frac{1}{\sqrt{3n+1}}$ for all integers greater than 1, which, together with a verification for the case where $n = 1$, establishes the weaker inequality we originally tried to prove using mathematical induction.

4. Use mathematical induction to show that a rectangular checkerboard with an even number of cells and two squares missing, one white and one black, can be covered by dominoes.