

# Tutorial Sheet 9

Oct 12,14,15

1. Solve the following recurrence relations

- (a)  $a_r^2 - 2a_{r-1}^2 = 1, a_0 = 2$
- (b)  $ra_r + ra_{r-1} - a_{r-1} = 2^r, a_0 = 273.$
- (c)  $a_r^2 - 2a_{r-1} = 0, a_0 = 4.$
- (d)  $a_r = \sqrt{a_{r-1} + \sqrt{a_{r-2} + \dots}}, a_0 = 4.$
- (e)  $a_r - ra_{r-1} = r!$

2. Solve the recurrence relation  $T(n) = nT^2(n/2)$  with initial condition  $T(1) = 6$  when  $n = 2^k$  for some integer  $k$ . [Hint: Let  $n = 2^k$  and then make the substitution  $a_k = \log T(2^k)$  to obtain a linear non-homogeneous recurrence relation.]

3. Show that the recurrence relation

$$f(n)a_n = g(n)a_{n-1} + h(n),$$

for  $n \geq 1$ , and with  $a_0 = C$ , can be reduced to a recurrence relation of the form

$$b_n = b_{n-1} + Q(n)h(n)$$

where  $b_n = g(n+1)Q(n+1)a_n$ , with  $Q(n) = (f(1)f(2) \dots f(n-1))/(g(1)g(2) \dots g(n))$ .

Use this to solve the original recurrence relation to obtain  $a_n = \frac{C + \sum_{i=1}^n Q(i)h(i)}{g(n+1)Q(n+1)}$ .

4. Use Q3 to solve the recurrence relation  $(n+1)a_n = (n+3)a_{n-1} + n$ , for  $n \geq 1$ , with  $a_0 = 1$ .

5. Show that if  $a_n = a_{n-1} + a_{n-2}$ ,  $a_0 = s$  and  $a_1 = t$ , where  $s$  and  $t$  are constants, then  $a_n = sf_{n-1} + tf_n$  for all positive integers  $n$  where  $f_n$  is the  $n^{th}$  fibonacci number.

6. Solve the simultaneous recurrence relations

$$a_n = 3a_{n-1} + 2b_{n-1}, \quad b_n = a_{n-1} + 2b_{n-1}$$

with  $a_0 = 1$  and  $b_0 = 2$ .