Tutorial Sheet 1

27, 29, 30 July

1. Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p)$ is a tautology.
2. Show that $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$ are logically equivalent.
3. The nth statement in a list of 100 statements is “Exactly n of the statements in this list are false.”
    a. What conclusions can you draw from these statements?
    b. Answer part (a) if the nth statement is “At least n of the statements in this list are false.”
    c. Answer part (b) assuming that the list contains 99 statements.
4. A collection of logical operators is called functionally complete if every compound proposition is logically equivalent to a compound proposition involving only these logical operators.
    a. Show that ¬, ∧, and ∨ form a functionally complete collection of logical operators.
    b. Show that ¬ and ∧ form a functionally complete collection of logical operators. [Hint: First use a De Morgan law to show that p ∨ q is logically equivalent to ¬(¬p ∧¬q).]
    c. Show that ¬ and ∨ form a functionally complete collection of logical operators.
5. How many different truth tables of compound propositions are there that involve the propositional variables p and q?
6. Teachers in the Middle Ages supposedly tested the realtime propositional logic ability of a student via a technique known as an obligato game. In an obligato game, a number of rounds is set and in each round the teacher gives the student successive assertions that the student must either accept or reject as they are given. When the student accepts an assertion, it is added as a commitment; when the student rejects an assertion its negation is added as a commitment. The student passes the test if the consistency of all commitments is maintained throughout the test.
    a. Suppose that in a three-round obligato game, the teacher first gives the student the proposition $p \rightarrow q$, then the proposition $\neg(p \vee r) \vee q$, and finally the proposition q. For which of the eight possible sequences of three answers will the student pass the test?
    b. Suppose that in a four-round obligato game, the teacher first gives the student the proposition $\neg(p \rightarrow (q \wedge r))$, then the proposition $p \vee \neg q$, then the proposition $\neg r$, and finally, the proposition $(p \wedge r) \vee (q \rightarrow p)$. For which of the 16 possible sequences of four answers will the student pass the test?
    c. Explain why every obligato game has a winning strategy.