Tutorial Sheet 1

1. Design DFA for the following languages over $\{0, 1\}$

   - The set of all strings such that every block of of five consecutive symbols have at least two 0's.
   - The set of all strings beginning with a 1 which interpreted as an integer is congruent to zero modulo 5.
   - The set of all strings with at most one pair of consecutive 0's and at most one pair of consecutive 1's.
   - The set of strings not containing the substring 110.
   - The set of all strings such that every substring of five consecutive symbols contains at least two 0s.
   - The set of strings $\{x0y|x, y \in \{0, 1\}^*, |y| = 4\}$.

2. Design NFA for the following languages

   - The set of strings over $\{0, 1\}$ such that some pair of 0's is seprated by a string of length $4i, i \geq 0$.
   - The set of strings over $\{a, b\}$ that have the same value when multiplied from left to right as from left to right. The rules of multiplication are $a \times a = b, b \times b = a, a \times b = b, b \times a = b$. Note that $((a \times b) \times b) = a$ and $(a \times (b \times b)) = b$, i.e. it is not associative.
   - The set of strings of the form $\{xwx^R|x, w$ are strings over 0,1 of non-zero length$\}$.

3. Let $L, M \subseteq \Sigma^*$ be languages over a nonempty finite alphabet Σ. For any $x, y \in \Sigma^*$, the shuffle of x and y is defined by induction as follows: $\epsilon \otimes y = \{y\}, x \otimes \epsilon = \{x\}, ax' \otimes by' = a.(x' \otimes y) \bigcup b.(x \otimes y')$, where $x = ax'$ and $y = by'$. It is extended to languages $L \otimes M$ as follows: $L \otimes M = \bigcup_{x \in L, y \in M} x \otimes y$. Prove that there exists a DFA for recognising $L \otimes M$ if there exist DFAs for recognising L and M.

4. Construct a DFA over the alphabet $\Sigma = \{0, 1\}$ which accepts the language $L = \{x \in \{0, 1\}^* | \nu_0(x) = \nu_1(x), \forall y \preceq x [0 \leq |\nu_0(y) - \nu_1(y)| \leq 1]\}$. Prove that your DFA accepts exactly the language L.
   ($\nu_\sigma(x)$: number of times $\sigma \in \Sigma$ occurs in x)