# Adversarial Search
## Chapter 5

Mausam

(Based on slides of Stuart Russell,
Andrew Parks, Henry Kautz,
Linda Shapiro, Diane Cook)


Calvin and HOBBES
by WATTERSON
HA HA! YOUR CLEVER LITTLE STRATEGY HAS TRAGICALLY BACKFIRED! LOOK AT THIS JUMP! WATCH AND WEEP, FUZZ FACE!
THAT'S YOUR MOVE, RIGHT? I GET TO GO NOW, RIGHT? IT'S TOO LATE FOR YOU TO CHANGE YOUR MIND, RIGHT?
NOT SO FAST... MY HAND'S STILL ON IT.
JUMP, JUMP, JUMP! I WIN!
YOU WIN?!? AAUGGHH! YOU WON LAST TIME! I HATE IT WHEN YOU WIN!
AARRGGH! MFF! GNNK! I HATE THIS GAME! I HATE THE WHOLE WORLD!!
AGHHH! WHAT A STUPID GAME
YOU MUST HAVE CHEATED! YOU MUST HAVE USED SOME SNEAKY, UNDERHANDED MIND-MELD TO MAKE ME LOSE! I HATE YOU!
I DIDN'T WANT TO PLAY THIS IDIOTIC GAME IN THE FIRST PLACE!
I KNEW YOU'D CHEAT! I KNEW YOU'D WIN!
OH! OOH! AARG!
© 1987 Universal Press Syndicate
AAAAAAAAAAAAAAAAAA
HACK PANT PANT
WATTERSON
LOOK, IT'S JUST A GAME.
I KNOW. YOU SHOULD SEE ME WHEN I LOSE IN REAL LIFE!

# Game Playing

Why do AI researchers study game playing?

1. It's a good reasoning problem, formal and nontrivial.

2. Direct comparison with humans and other computer programs is easy.

# What Kinds of Games?

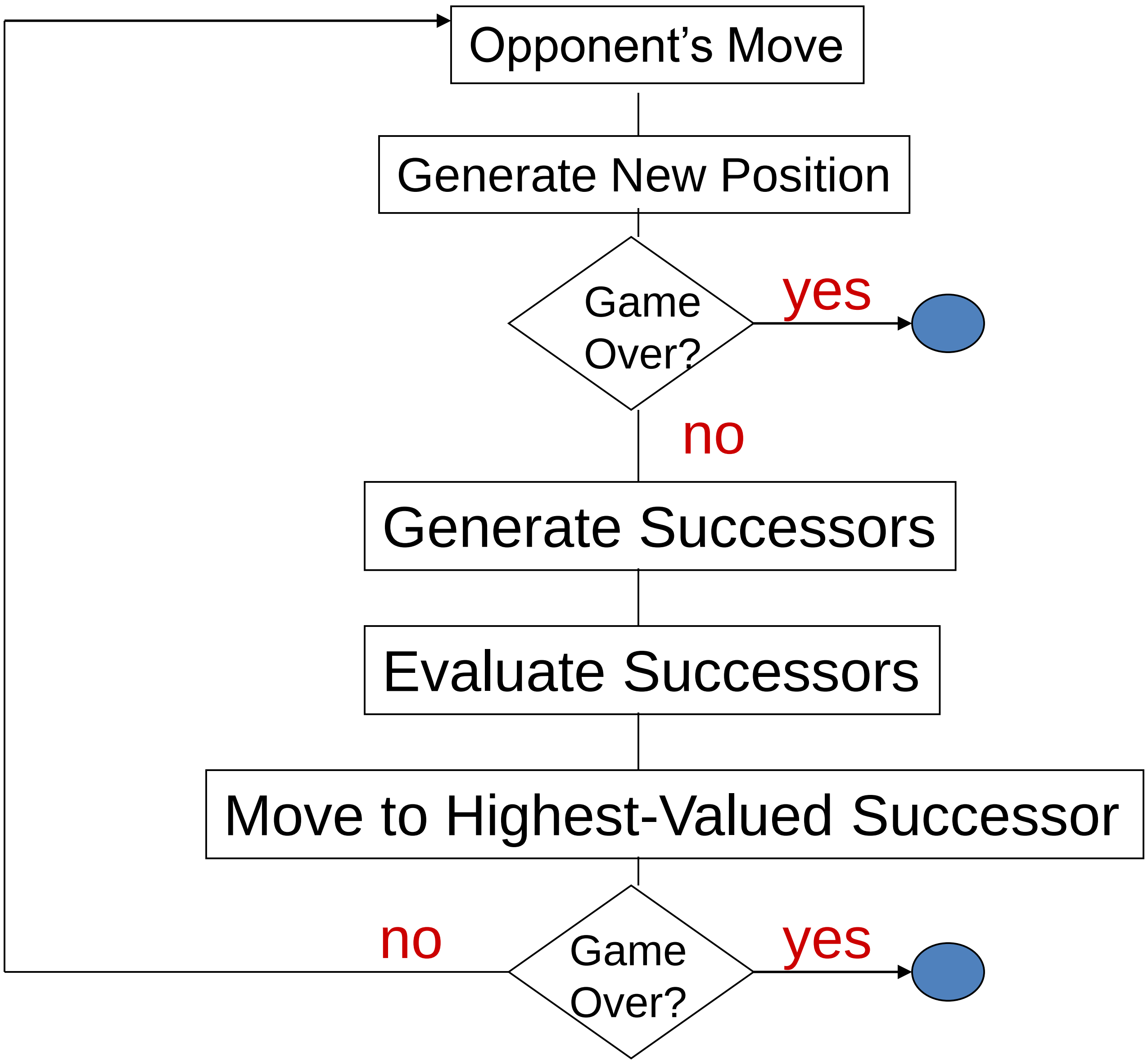
Mainly games of strategy with the following characteristics:

1. Sequence of moves to play
2. Rules that specify possible moves
3. Rules that specify a payment for each move
4. Objective is to maximize your payment

# Games vs. Search Problems

- Unpredictable opponent → specifying a move for every possible opponent reply
- Time limits → unlikely to find goal, must approximate

# Two-Player Game



# Games as Adversarial Search

- States:
  - board configurations
- Initial state:
  - the board position and which player will move
- Successor function:
  - returns list of (move, state) pairs, each indicating a legal move and the resulting state
- Terminal test:
  - determines when the game is over
- Utility function:
  - gives a numeric value in terminal states

  (e.g., -1, 0, +1 for loss, tie, win)

# Game Tree (2-player, Deterministic, Turns)

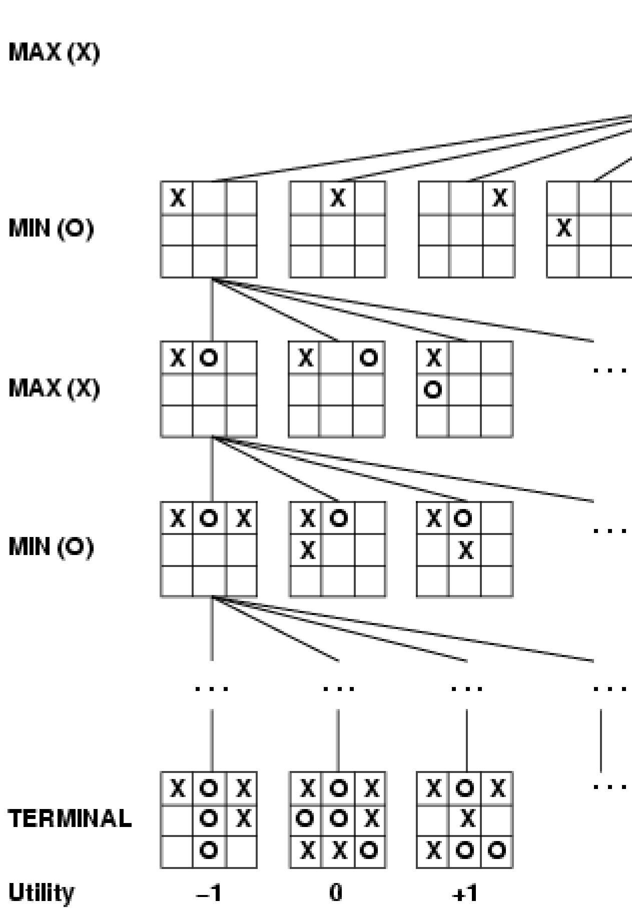
computer's turn

opponent's turn

computer's turn

opponent's turn

leaf nodes are evaluated




The computer is **Max**.
The opponent is **Min**.

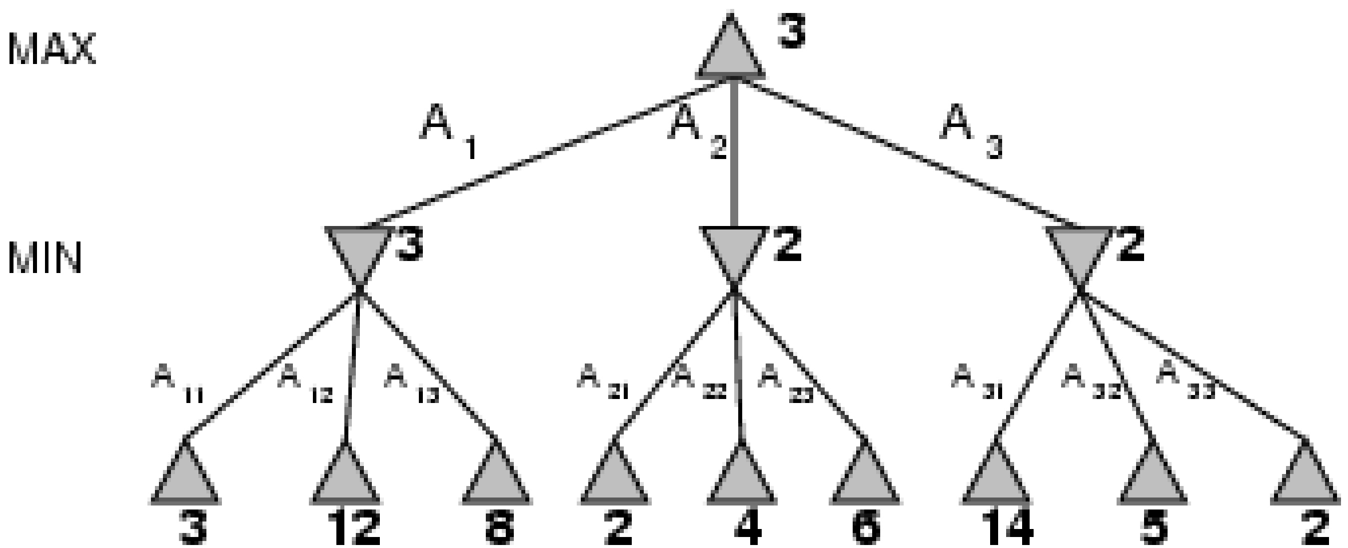
At the leaf nodes, the **utility function** is employed. Big value means good, small is bad.

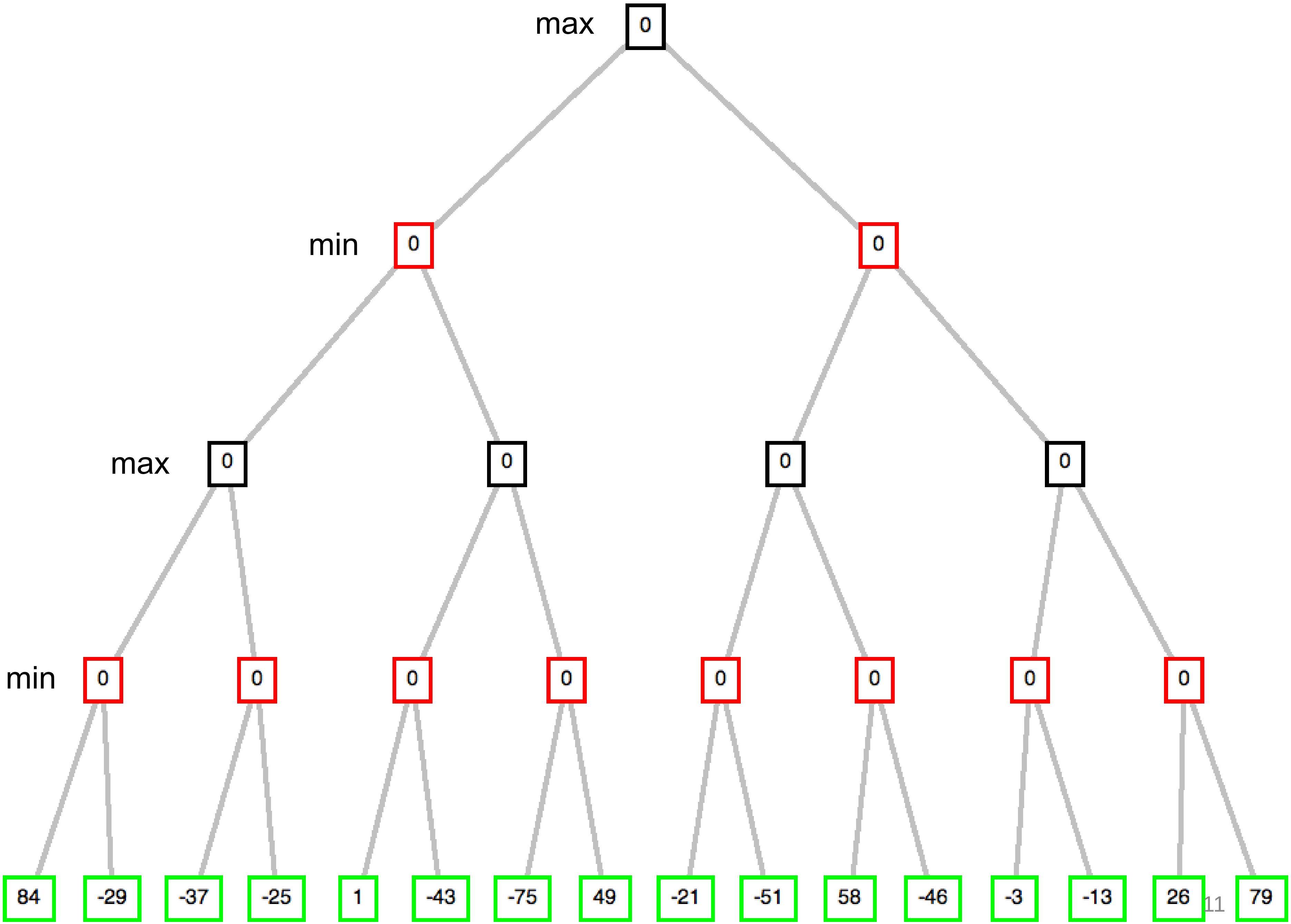
# Mini-Max Terminology

- move: a move by both players
- ply: a half-move
- utility function: the function applied to leaf nodes
- backed-up value
  - of a max-position: the value of its largest successor
  - of a min-position: the value of its smallest successor
- minimax procedure: search down several levels; at the bottom level apply the utility function, back-up values all the way up to the root node, and that node selects the move.

# Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
- E.g., 2-ply game:





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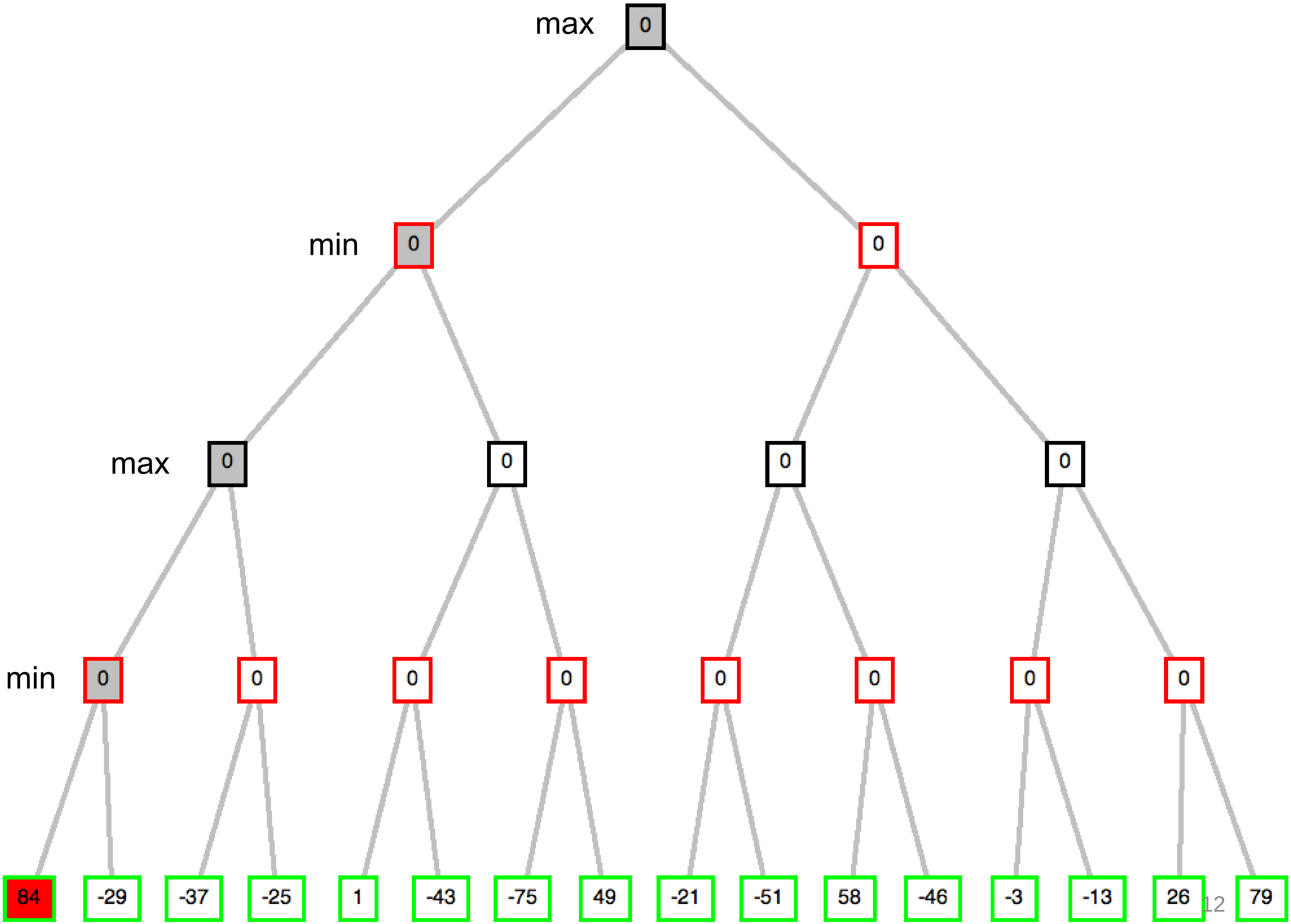
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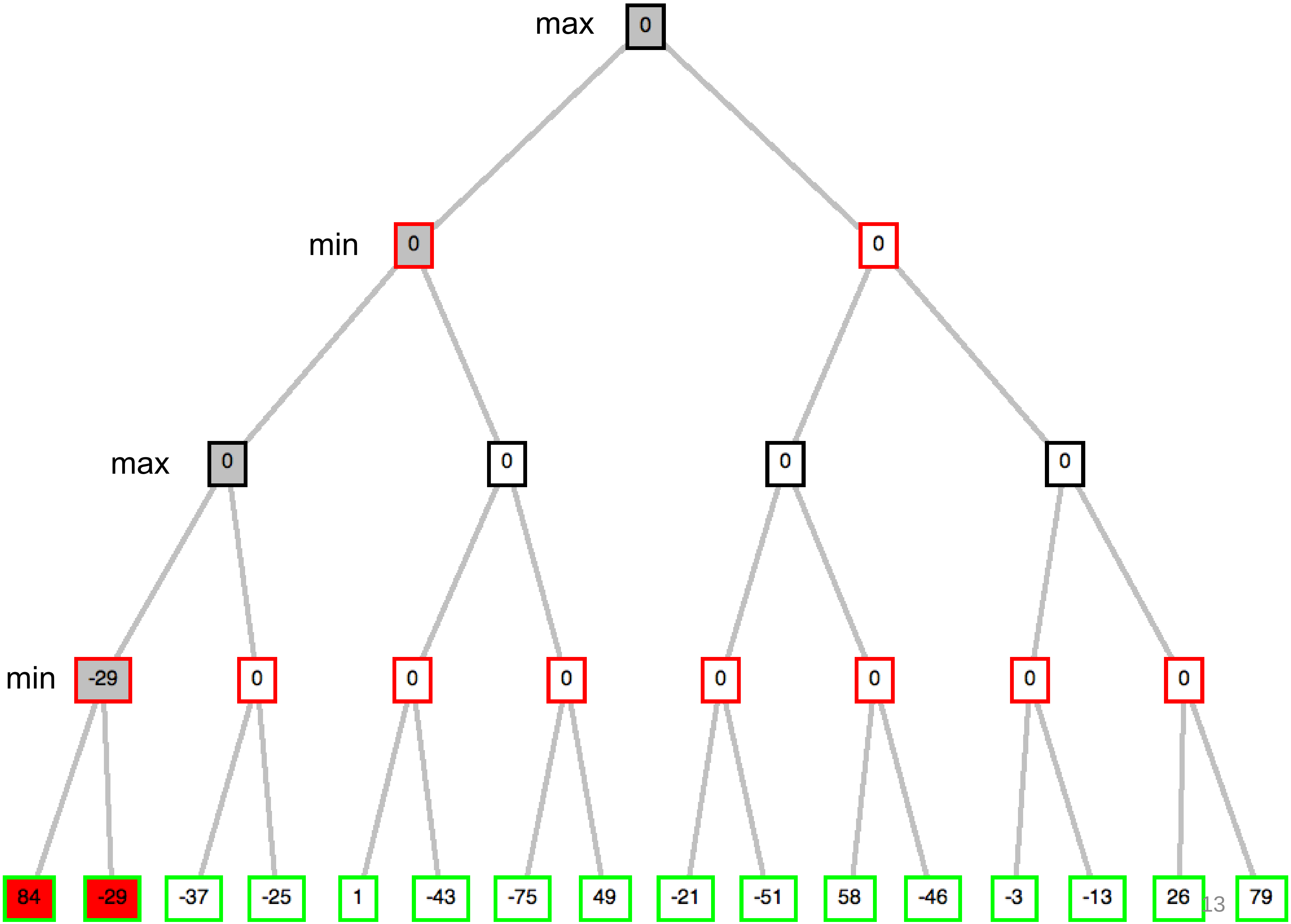
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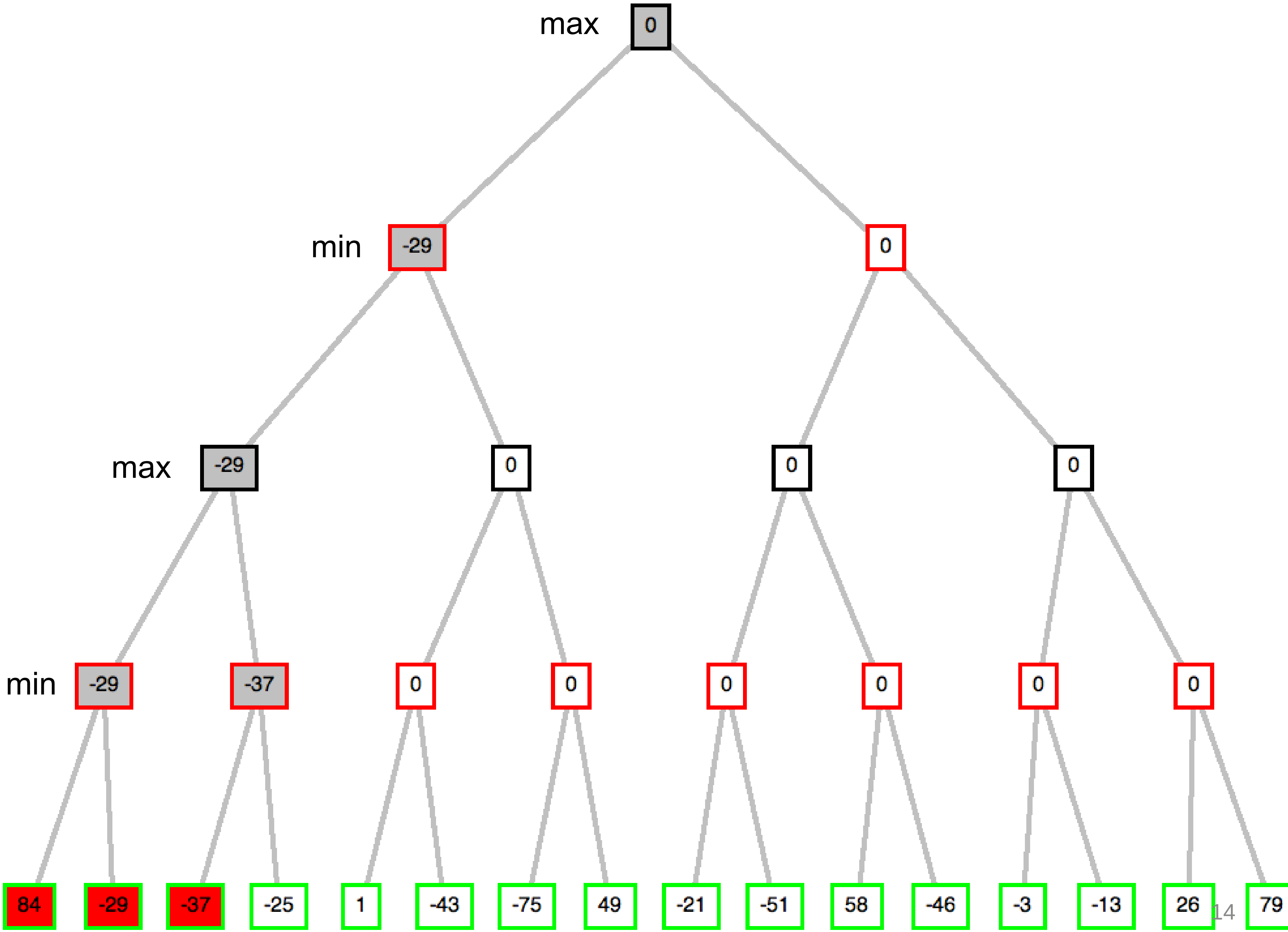
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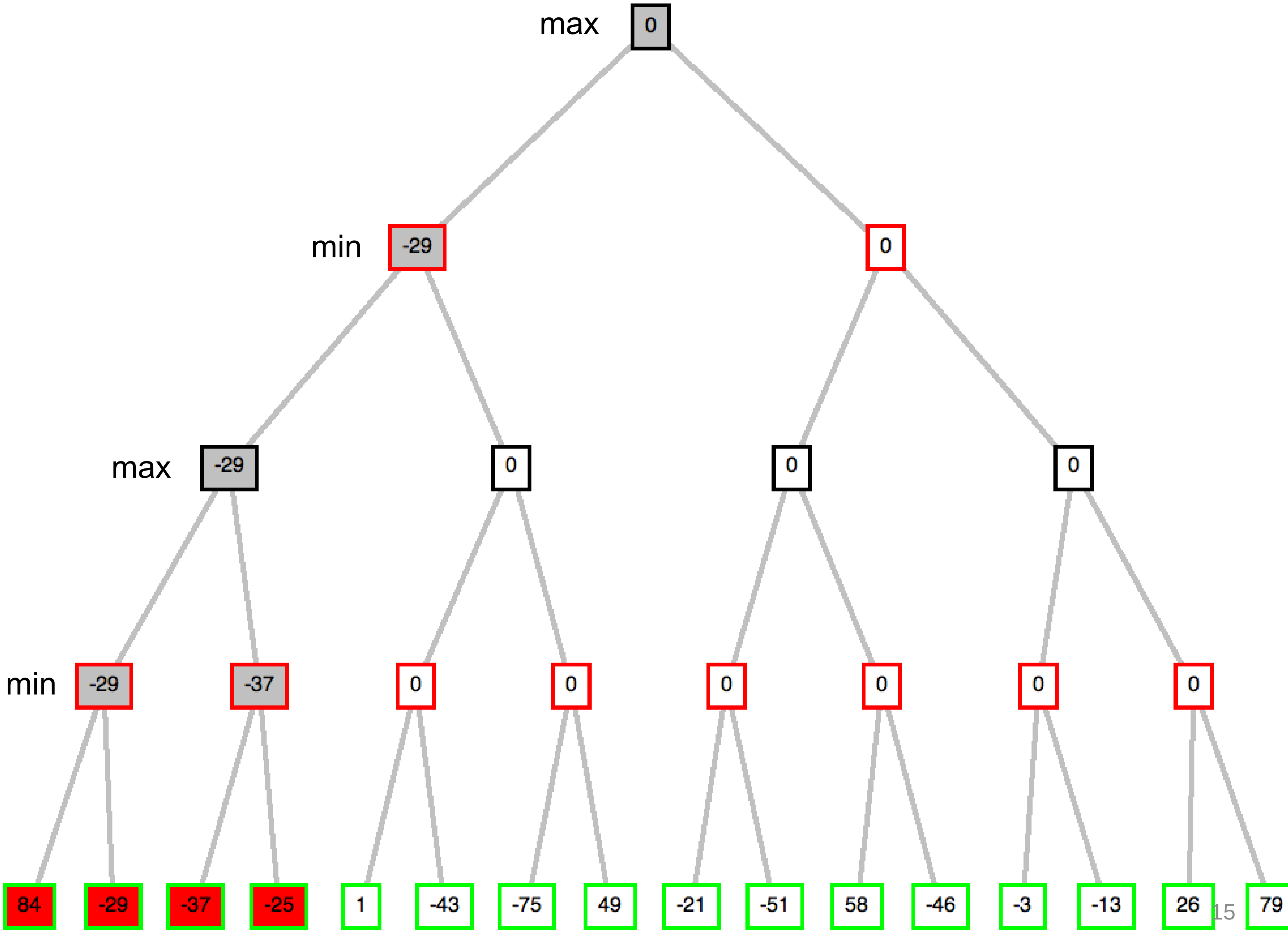
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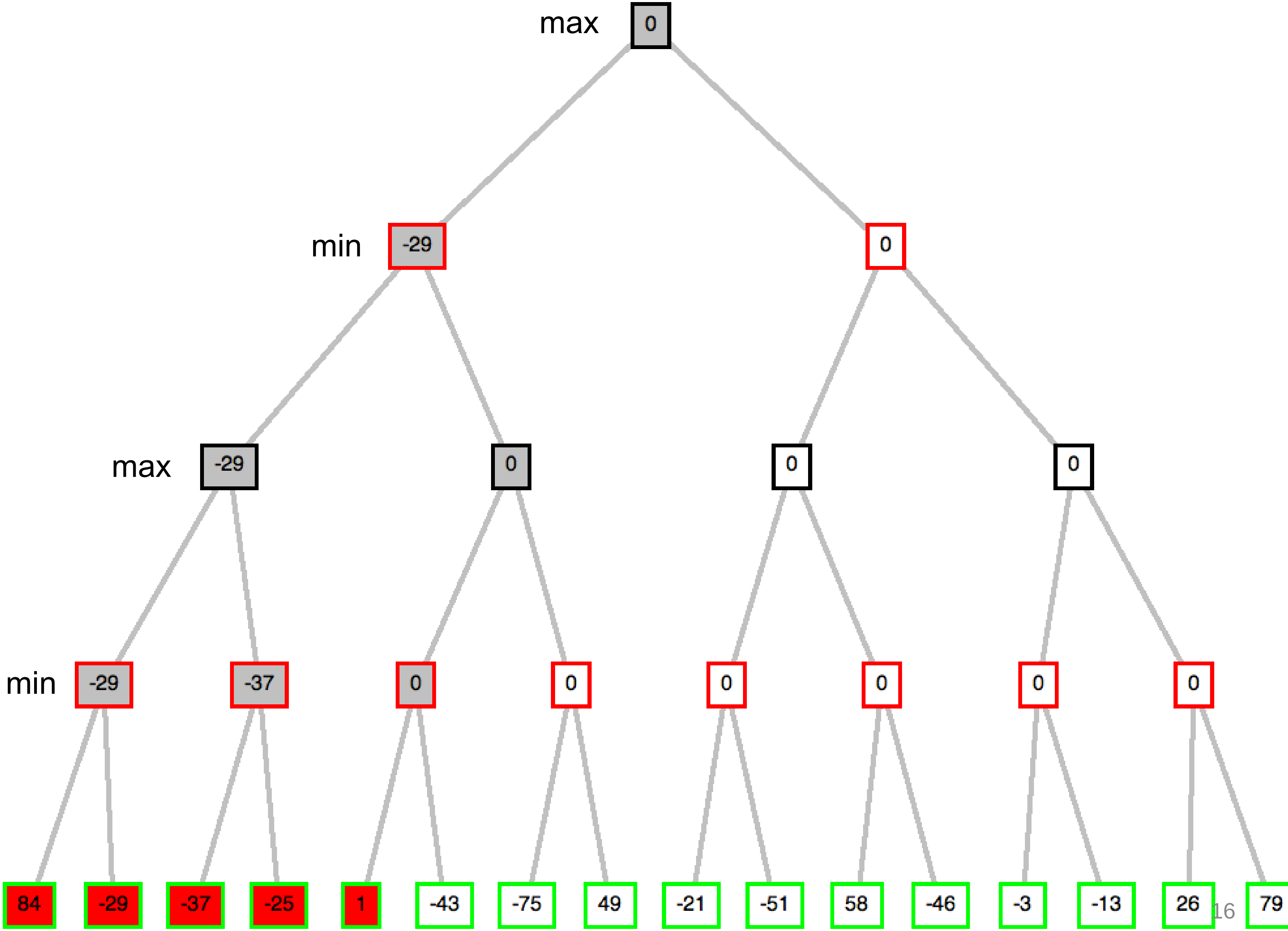
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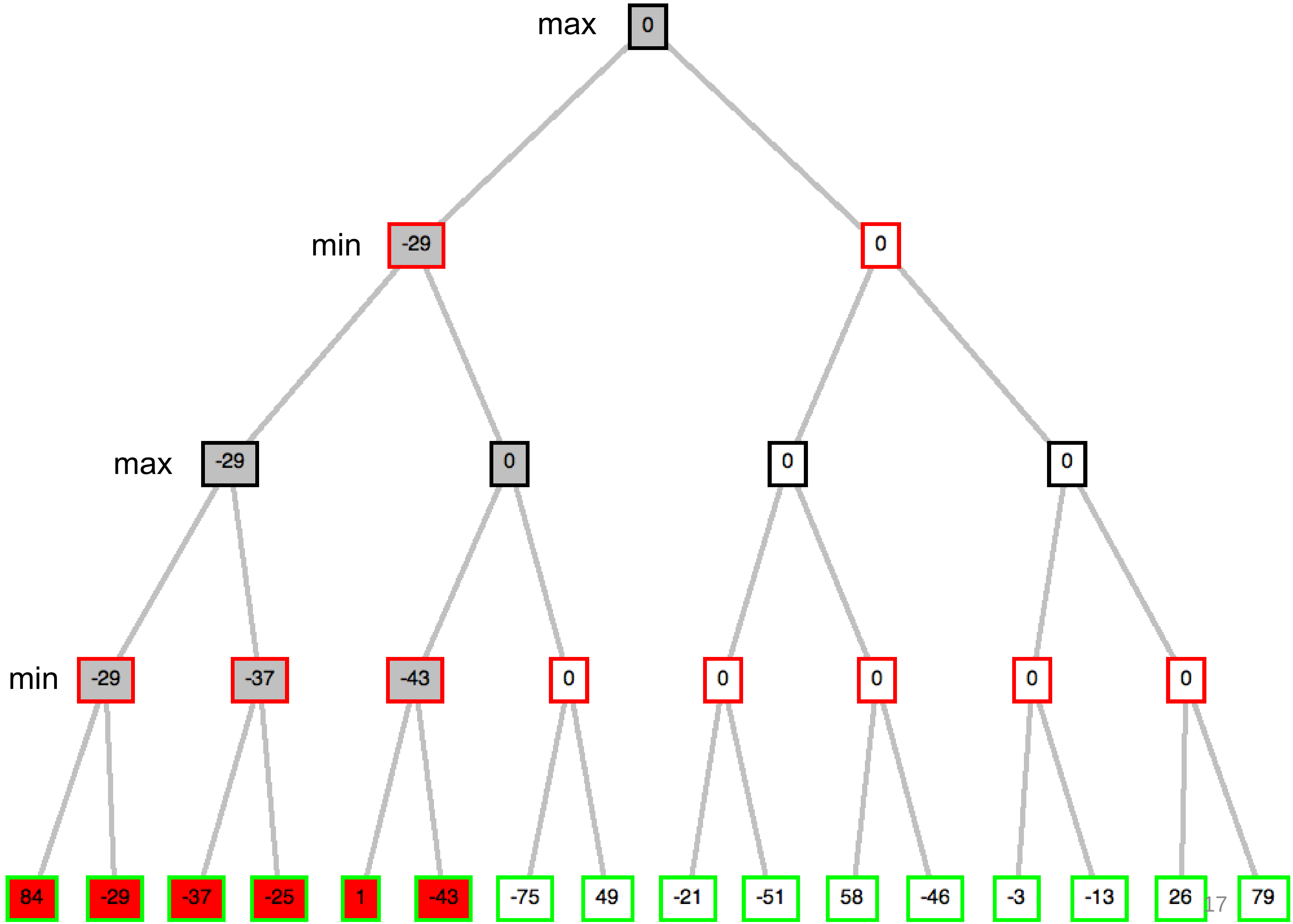
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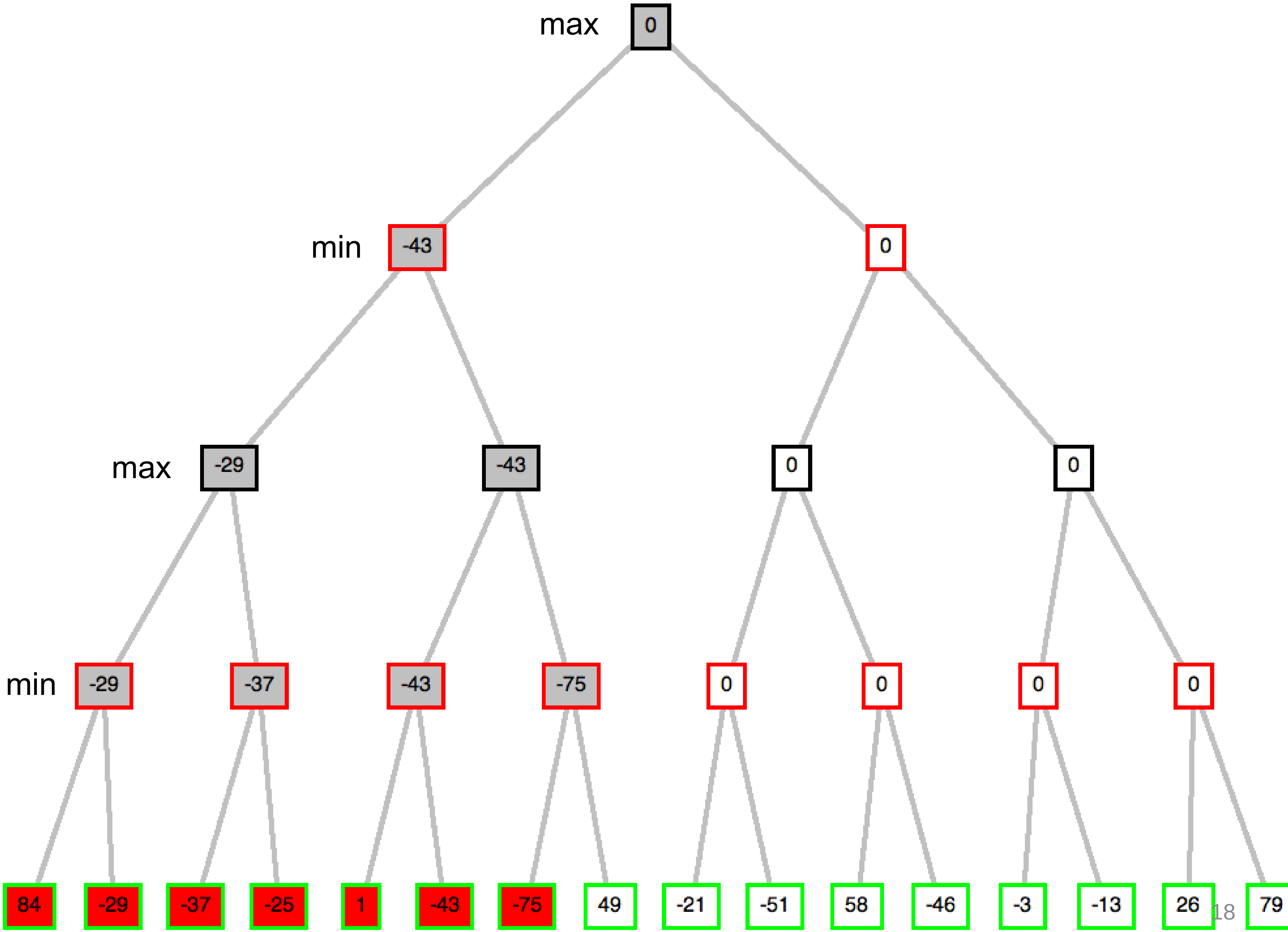
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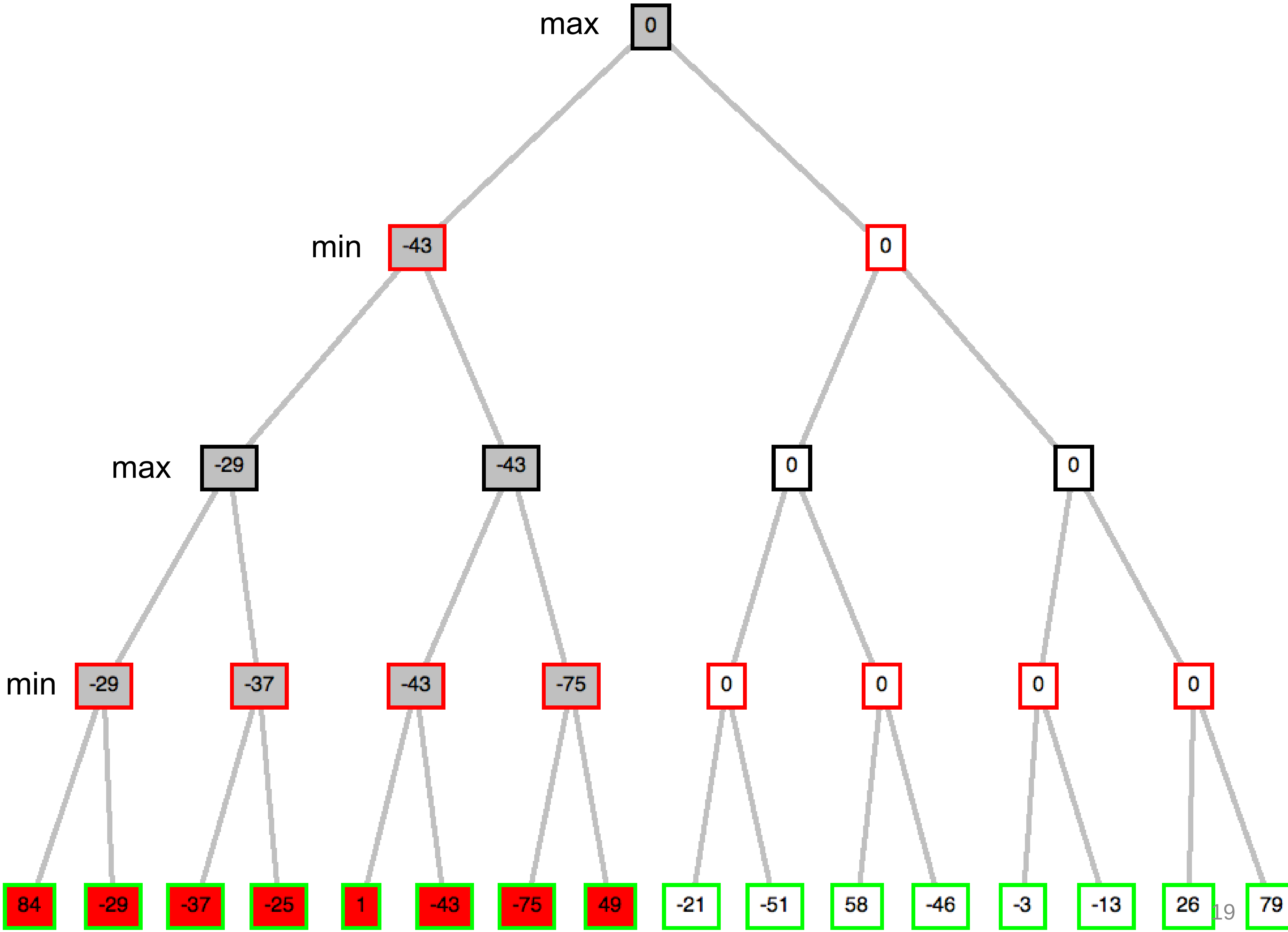
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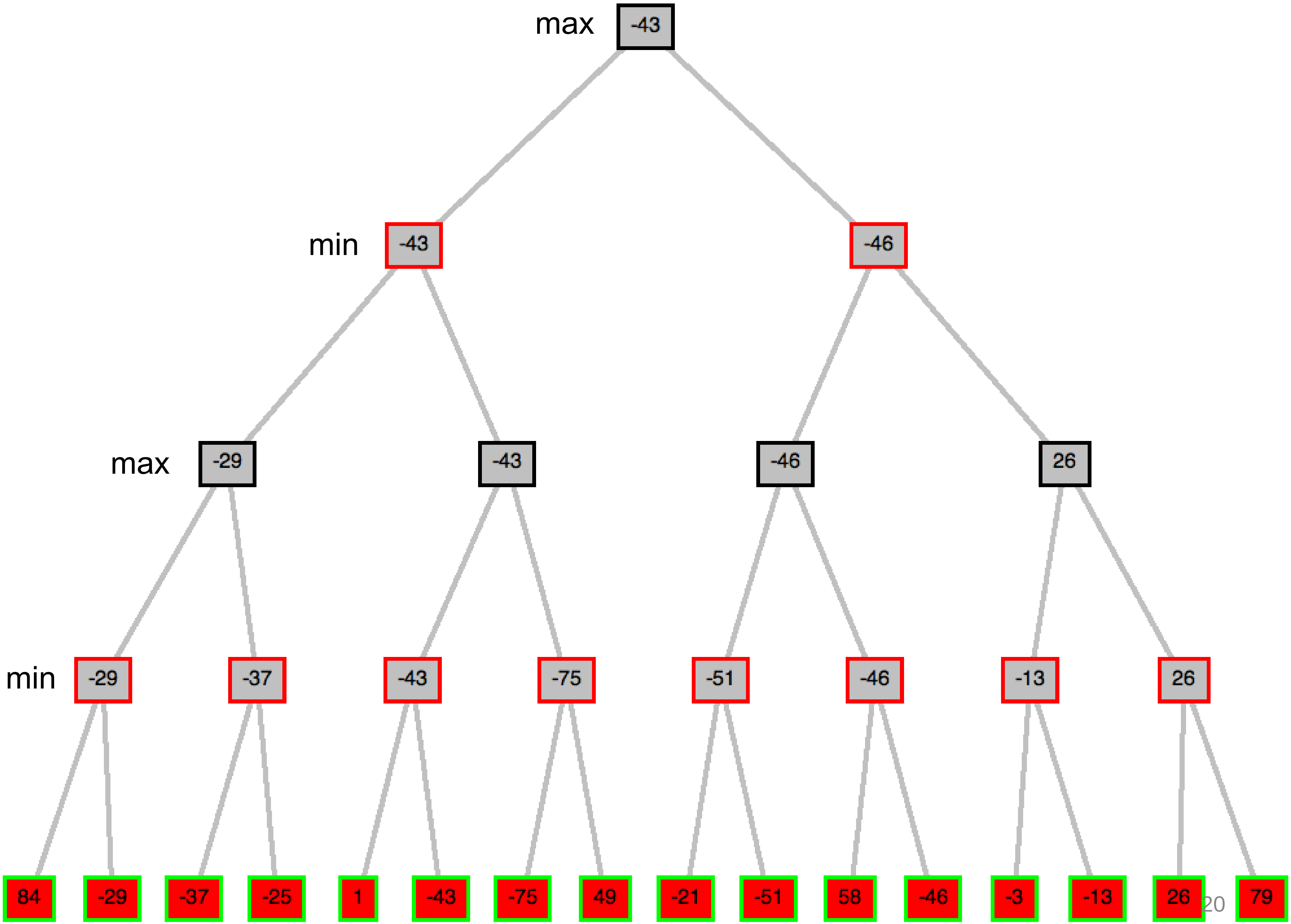
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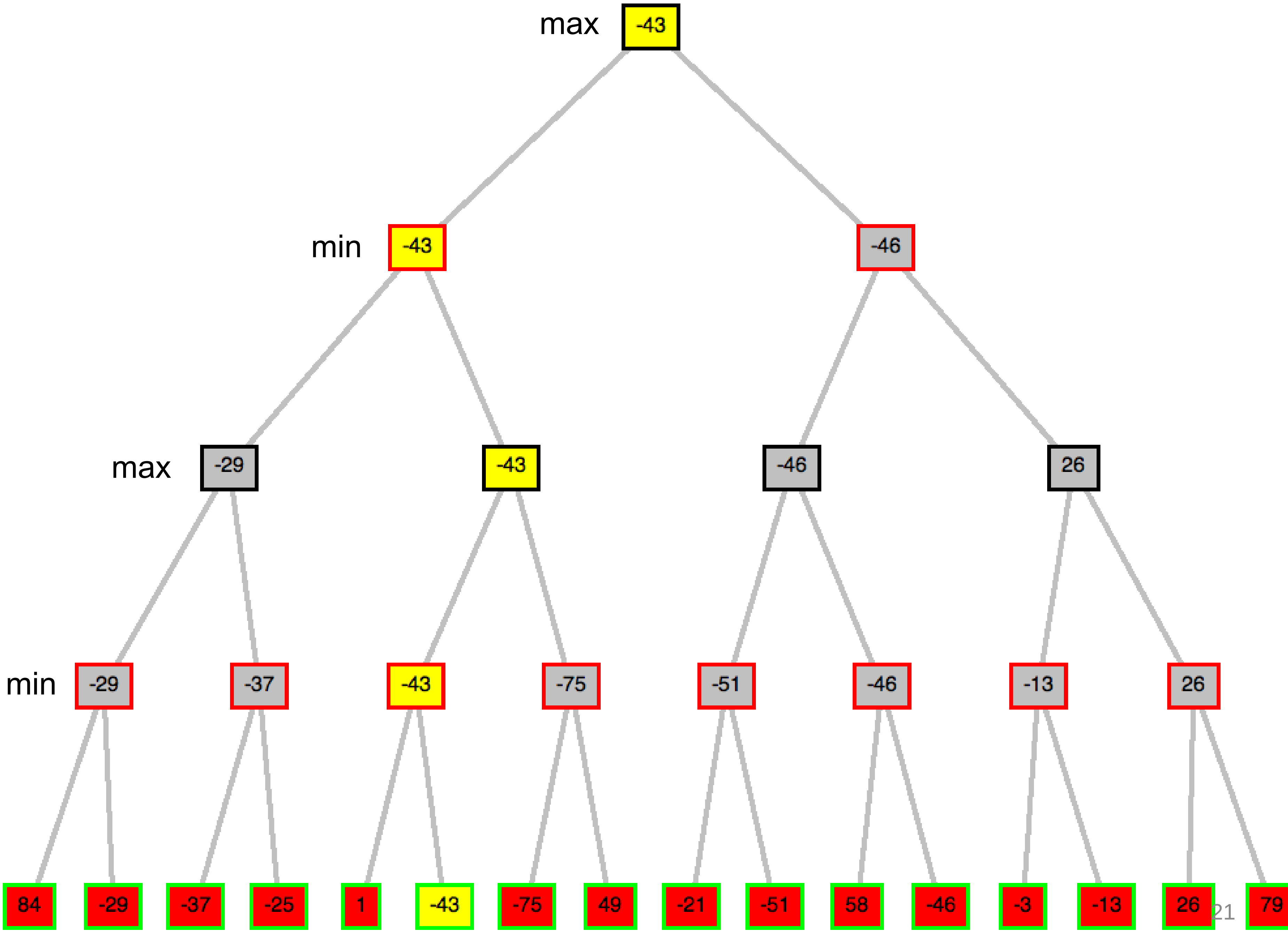
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# Minimax Strategy

- Why do we take the min value every other level of the tree?
- These nodes represent the opponent's choice of move.
- The computer assumes that the human will choose that move that is of least value to the computer.

# Minimax algorithm
# Adversarial analogue of DFS

```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

# Properties of Minimax

- Complete?
  - Yes (if tree is finite)
- Optimal?
  - Yes (against an optimal opponent)
  - No (does not exploit opponent weakness against suboptimal opponent)
- Time complexity?
  - $O(b^m)$
- Space complexity?
  - $O(bm)$ (depth-first exploration)

# Good Enough?

- Chess:
  - branching factor b≈35
  - game length m≈100
  - search space $b^m \approx 35^{100} \approx 10^{154}$
- The Universe:
  - number of atoms $\approx 10^{78}$
  - age $\approx 10^{18}$ seconds
  - $10^8$ moves/sec x $10^{78}$ x $10^{18} = 10^{104}$
- Exact solution completely infeasible

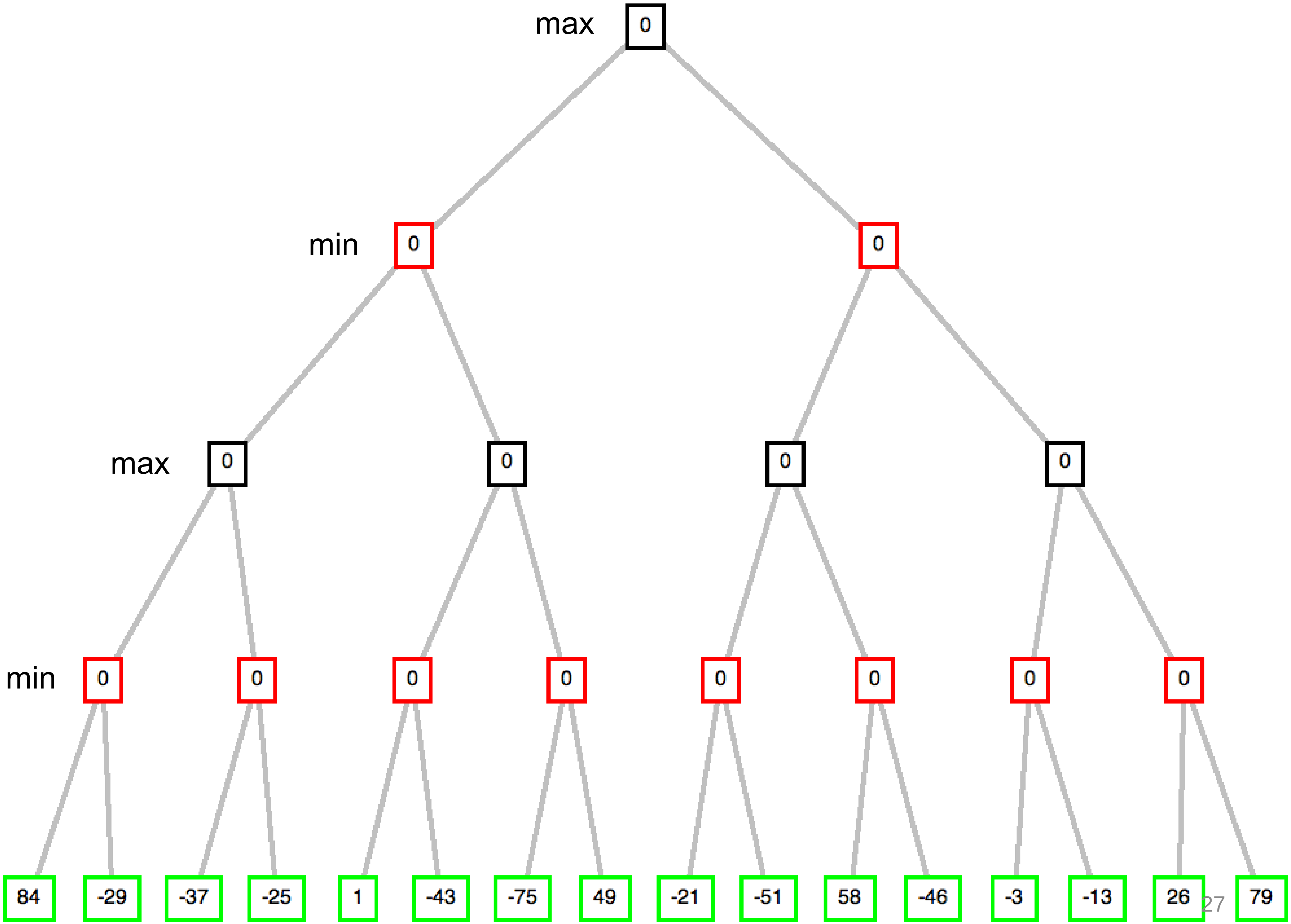
# Alpha-Beta Procedure

- The alpha-beta procedure can speed up a depth-first minimax search.
- Alpha: a lower bound on the value that a max node may ultimately be assigned

$$v \geq \alpha$$

- Beta: an upper bound on the value that a minimizing node may ultimately be assigned

$$v \leq \beta$$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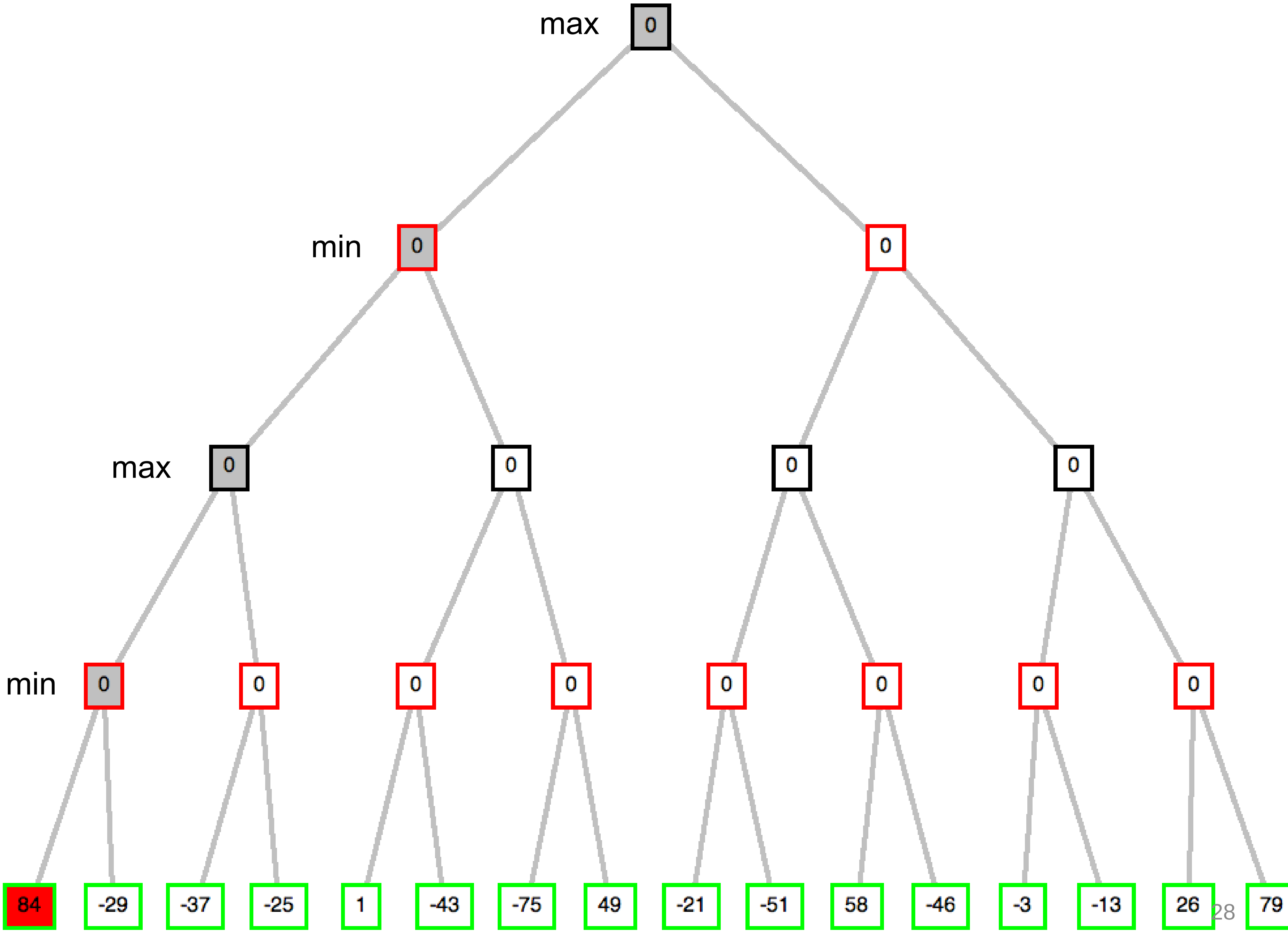
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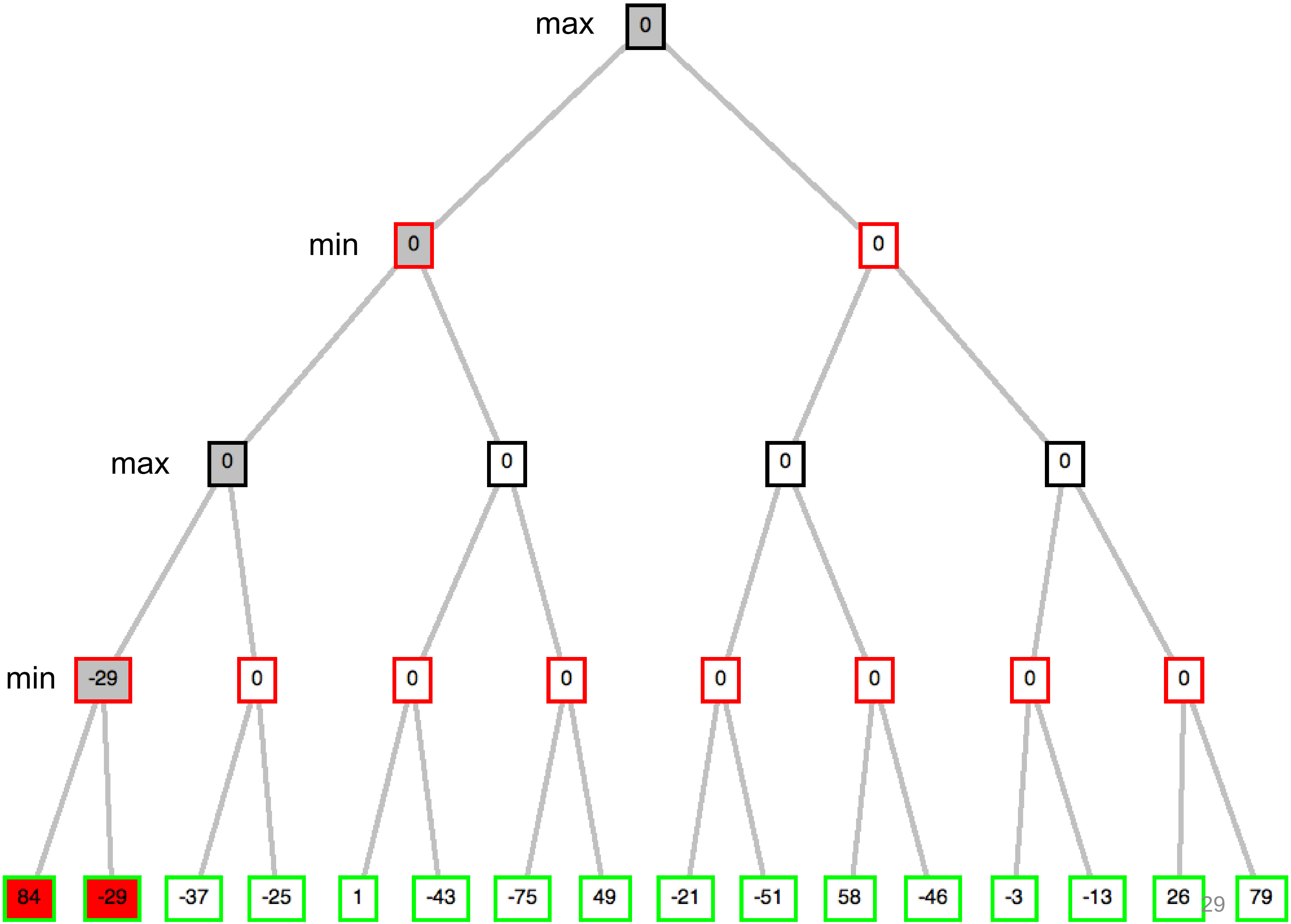
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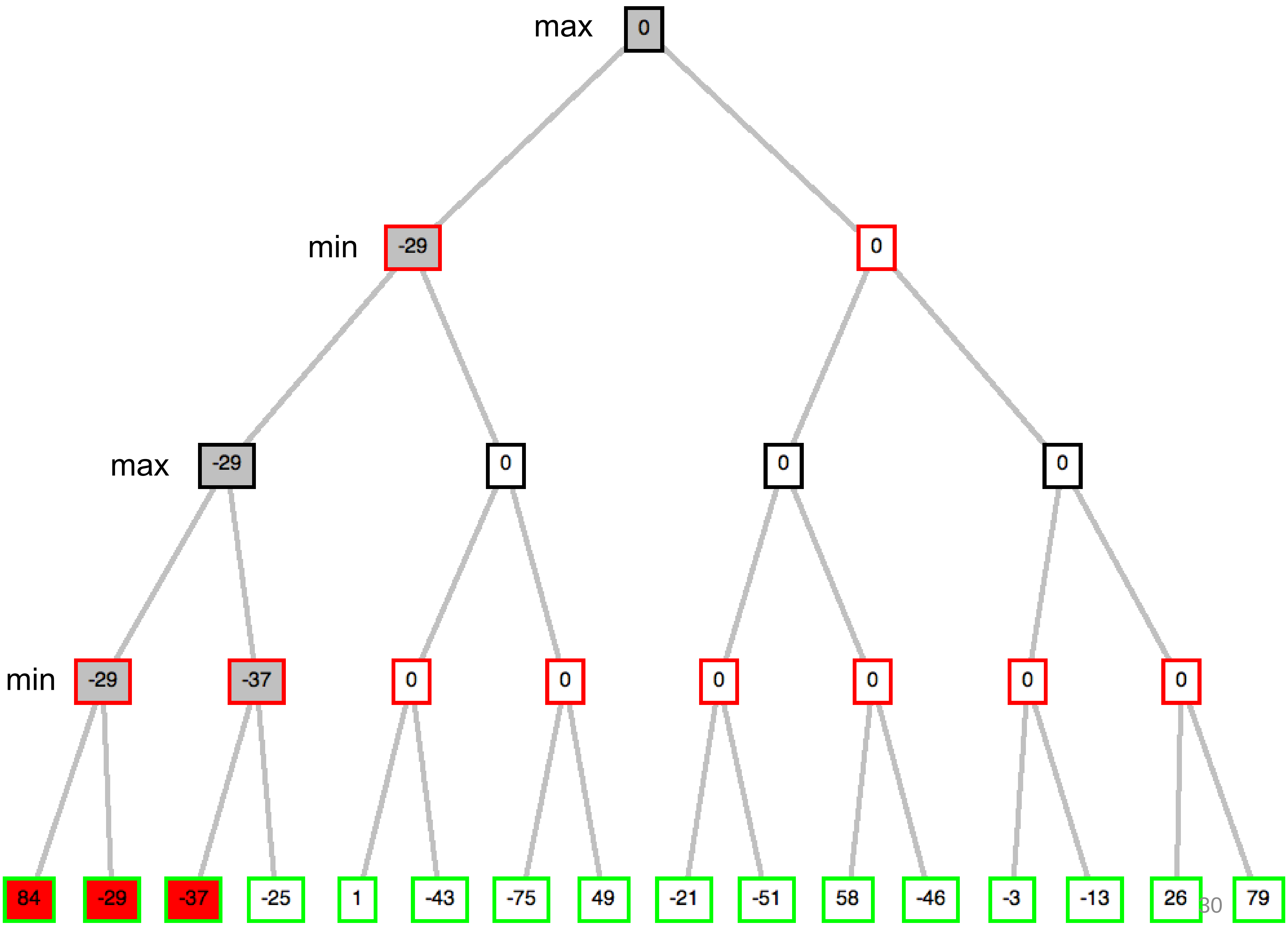
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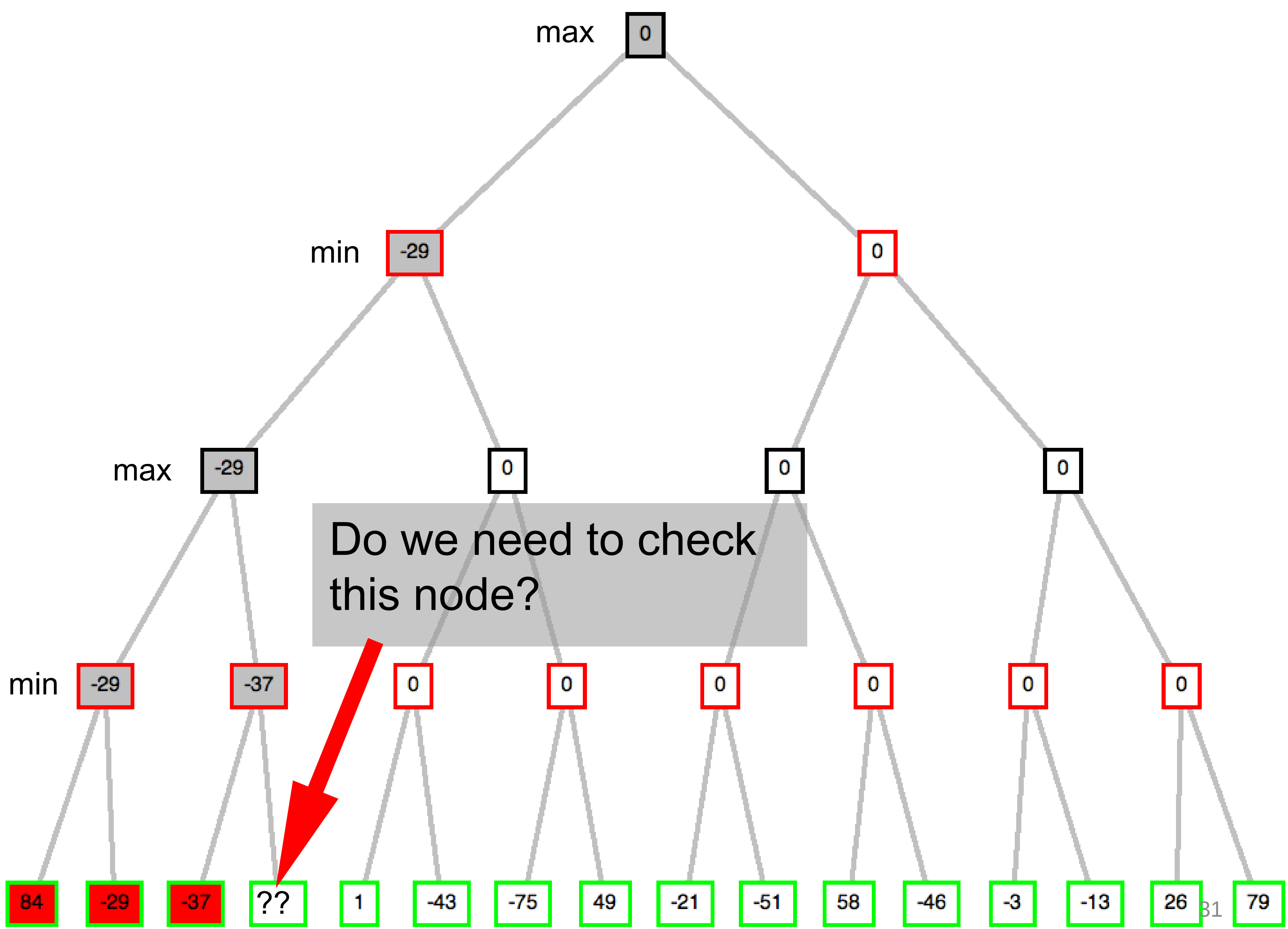
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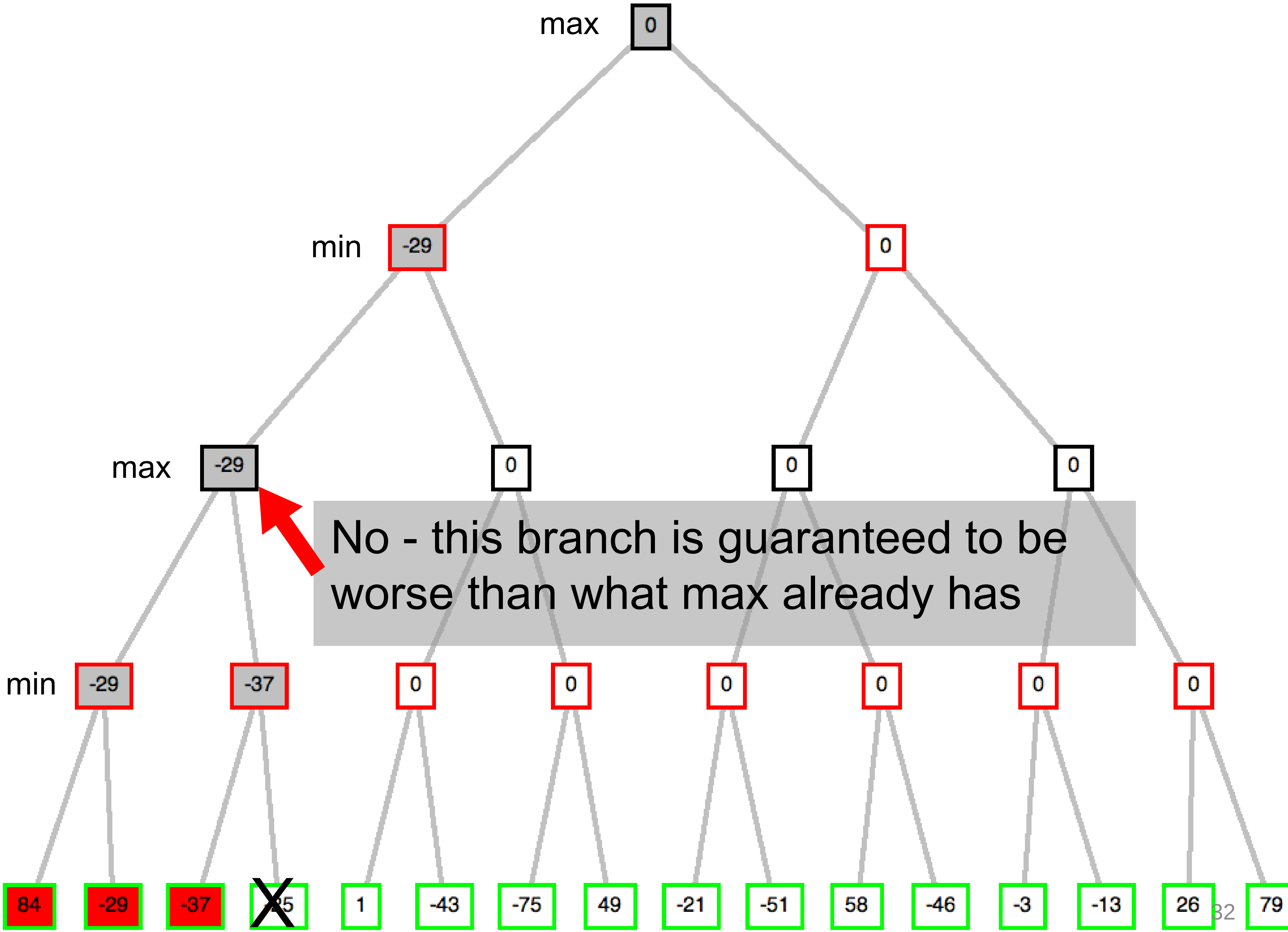
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# Alpha-Beta

```
MinVal(state, alpha, beta){
  if (terminal(state))
          return utility(state);
  for (s in children(state)){
     child = MaxVal(s,alpha,beta);
     beta = min(beta,child);
     if (alpha>=beta) return child;
  }
  return best child (min); }
```

**alpha** = the **highest** value for **MAX** along the path

**beta** = the **lowest** value for **MIN** along the path

# Alpha-Beta

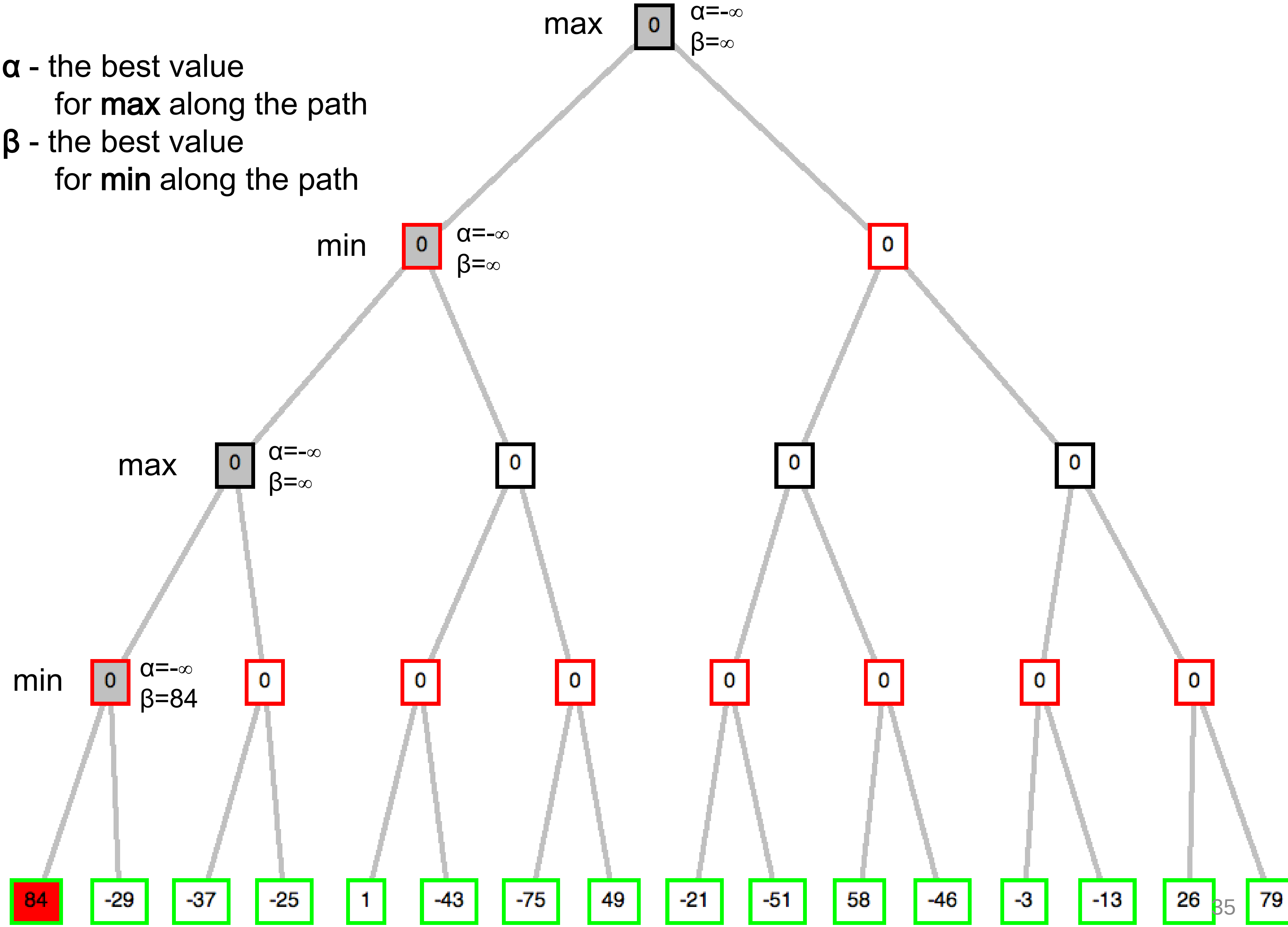
```
MaxVal(state, alpha, beta){
  if (terminal(state))
           return utility(state);
  for (s in children(state)){
     child = MinVal(s,alpha,beta);
     alpha = max(alpha,child);
     if (alpha>=beta) return child;
  }
  return best child (max); }
```

**alpha** = the **highest** value for **MAX** along the path

**beta** = the **lowest** value for **MIN** along the path

**α** **-** the best value for **max** along the path

**β** **-** the best value for **min** along the path



**α** - the best value
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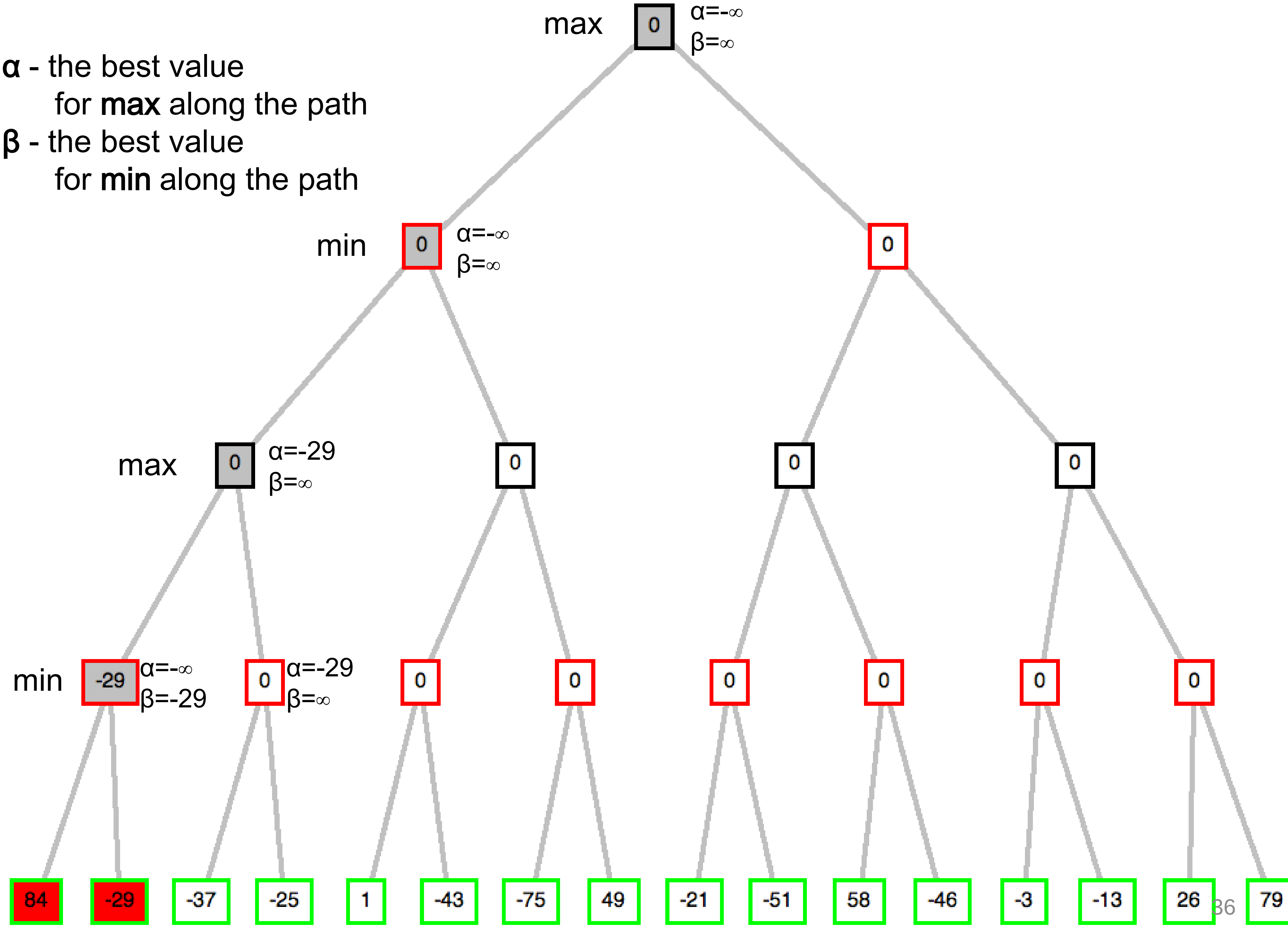

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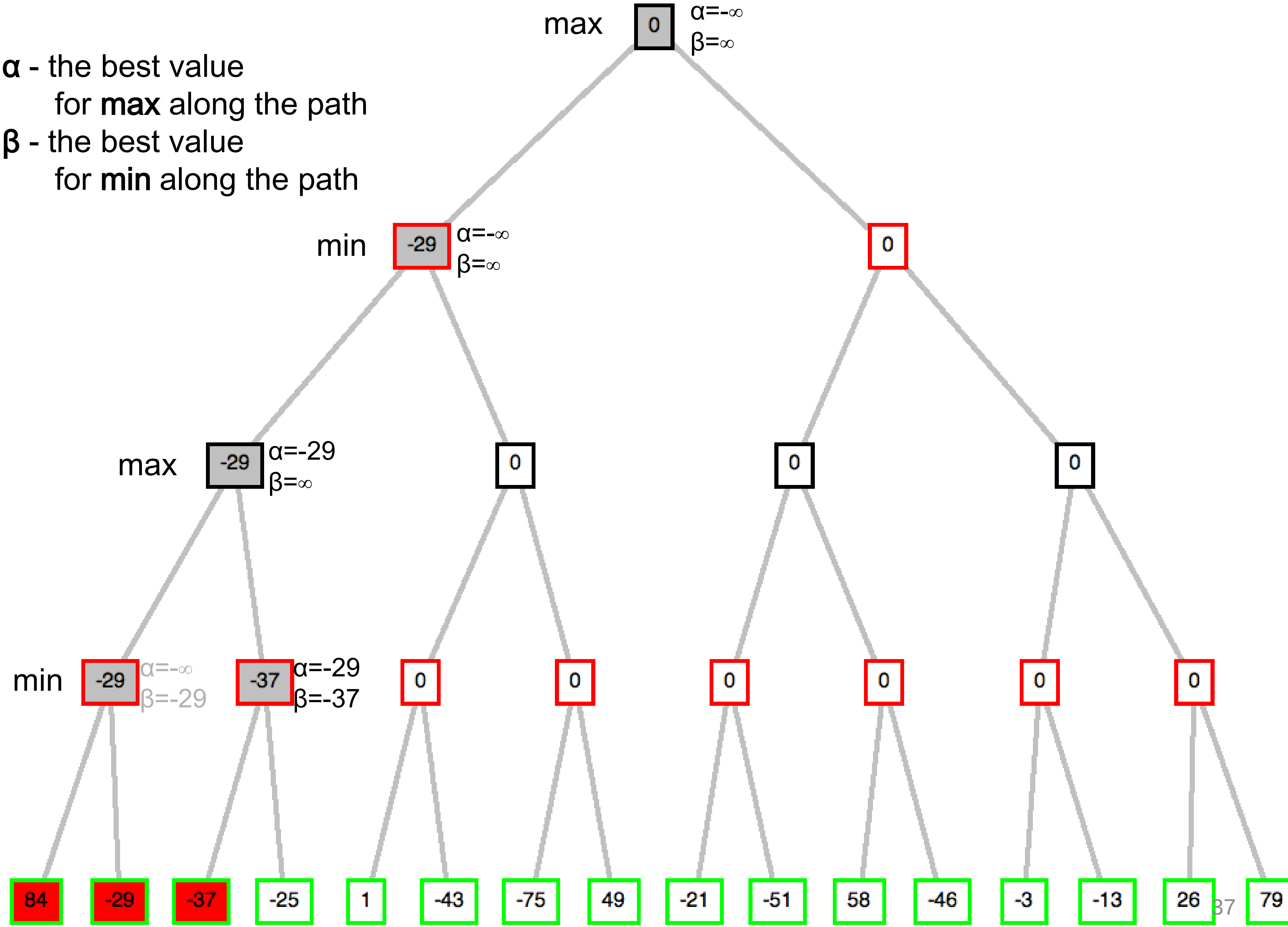

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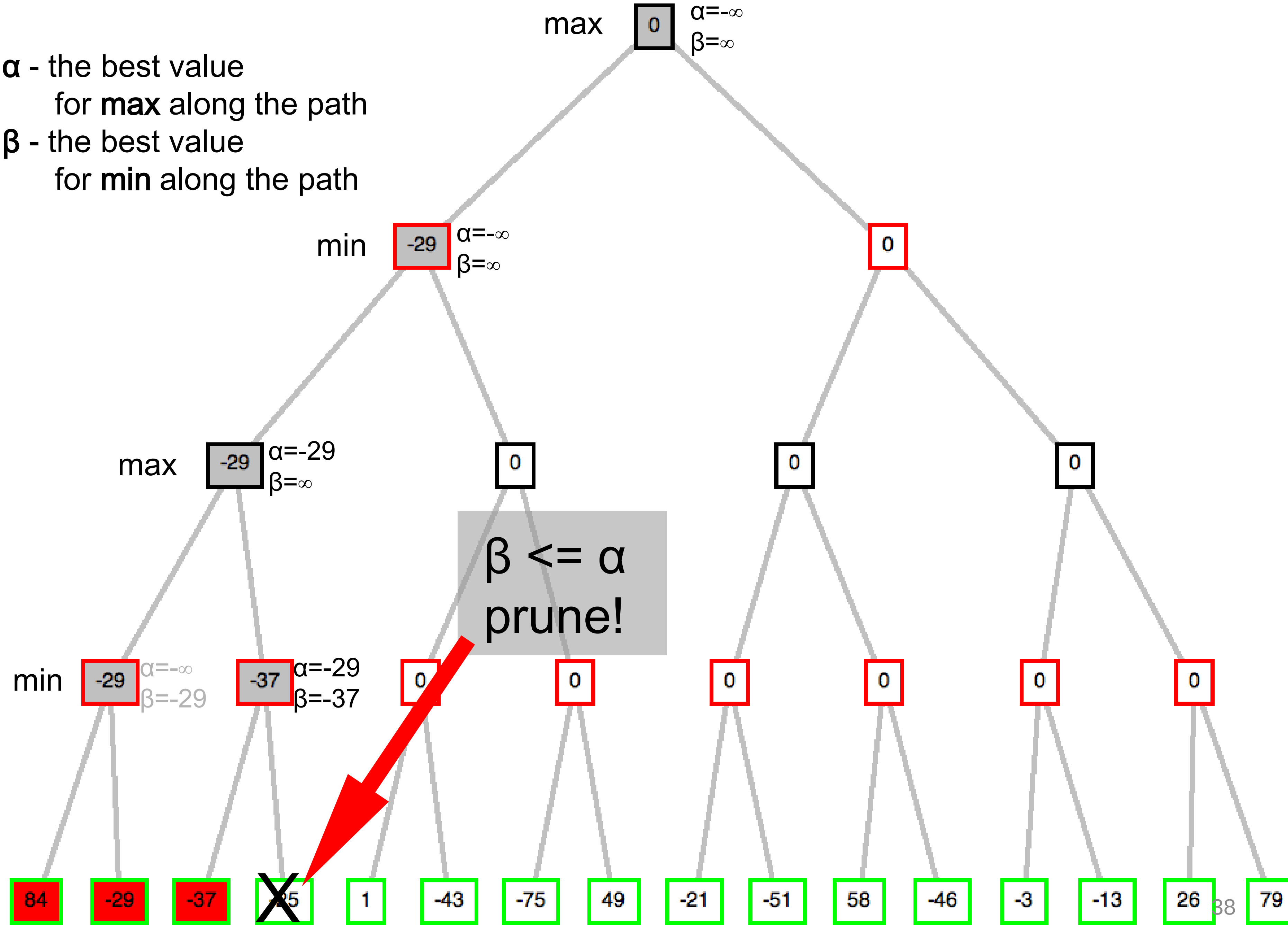

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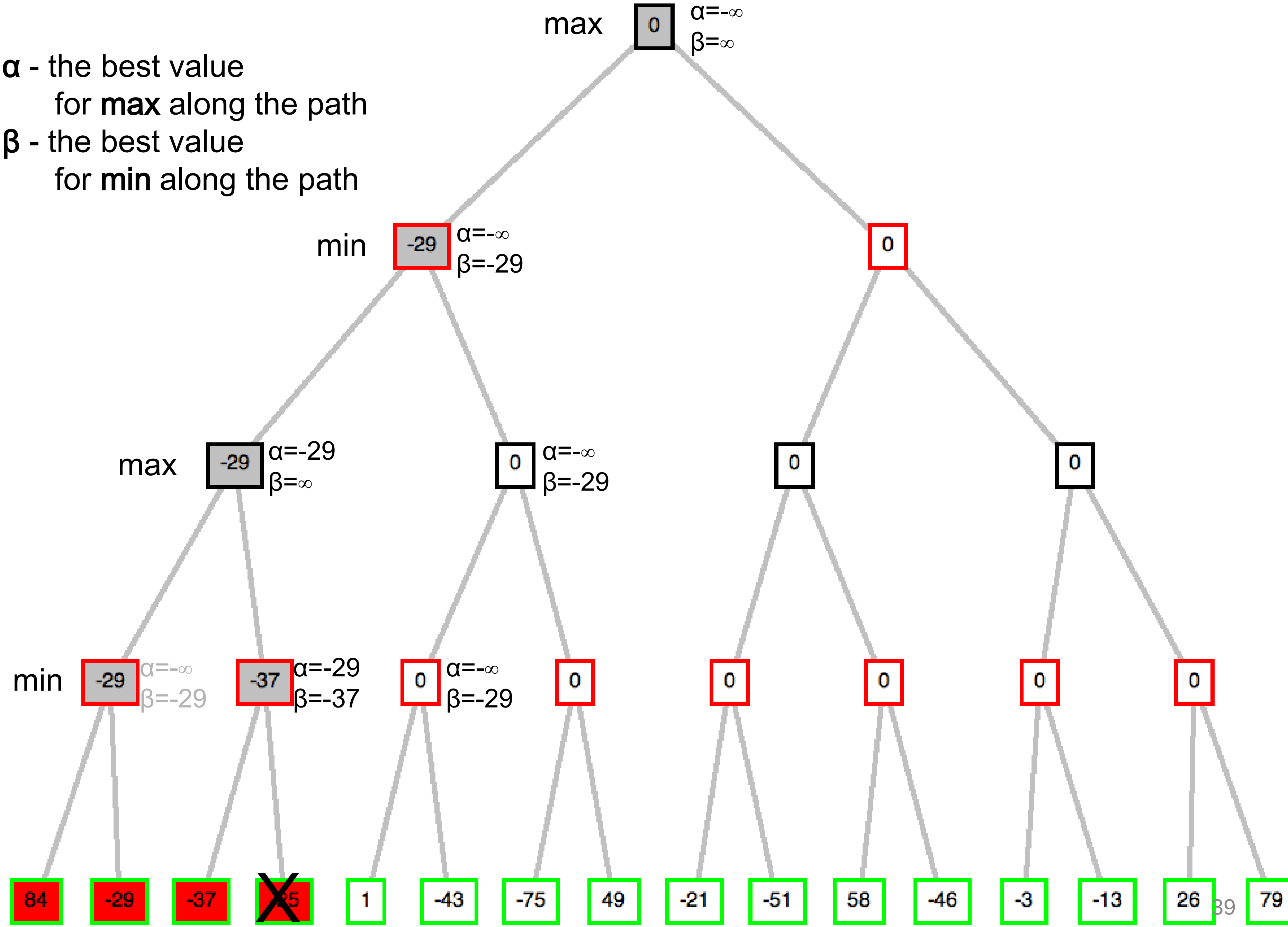

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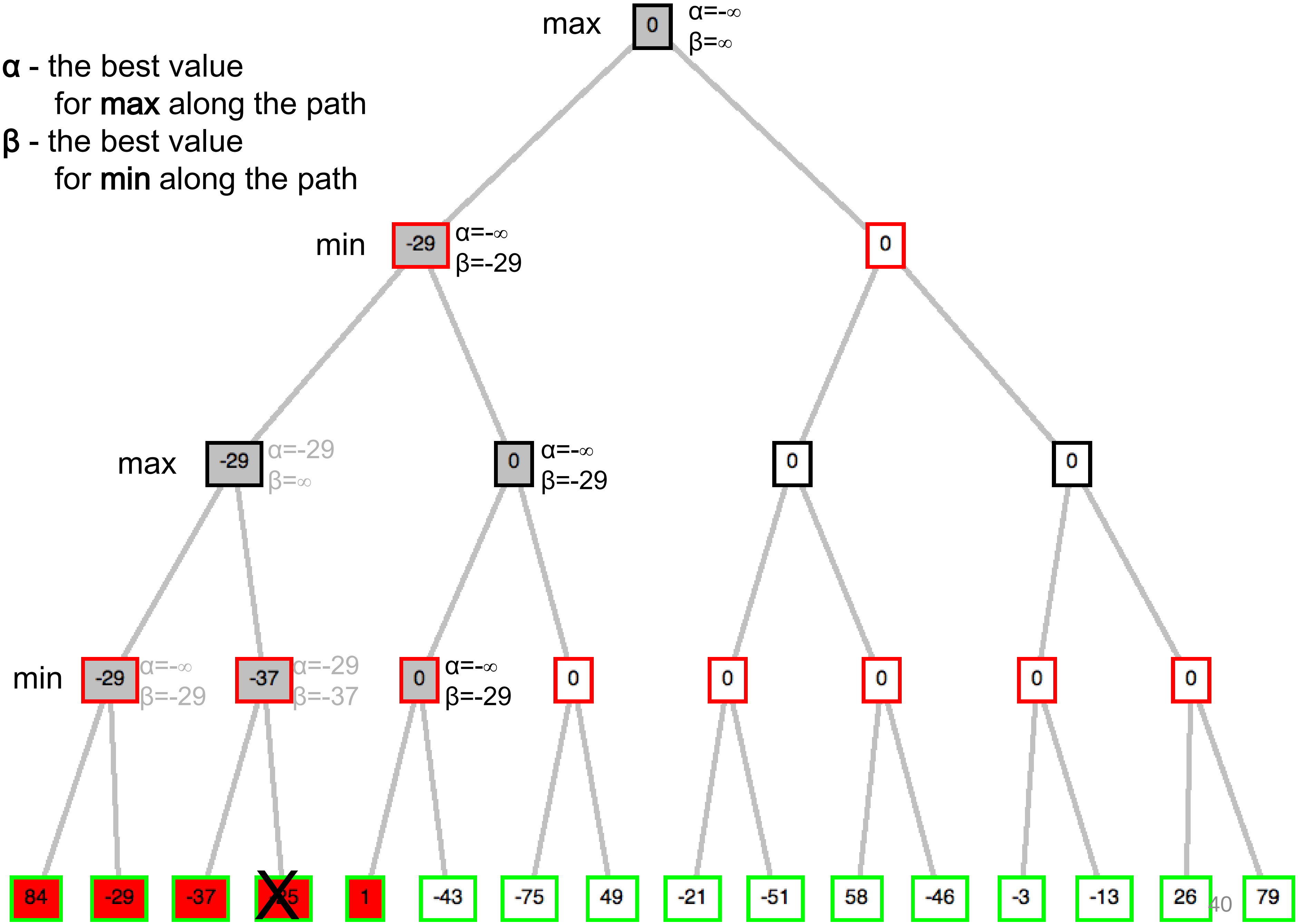

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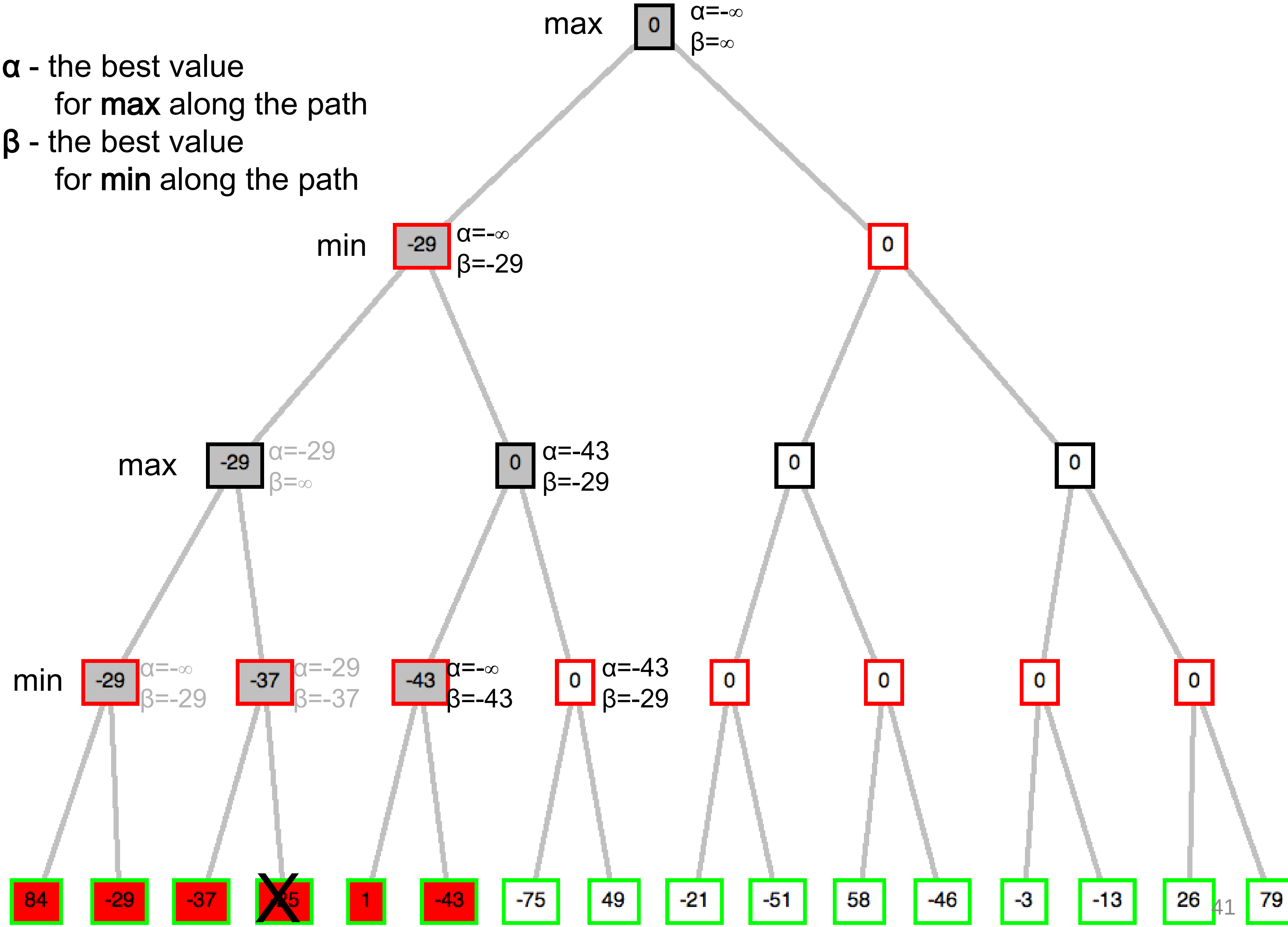

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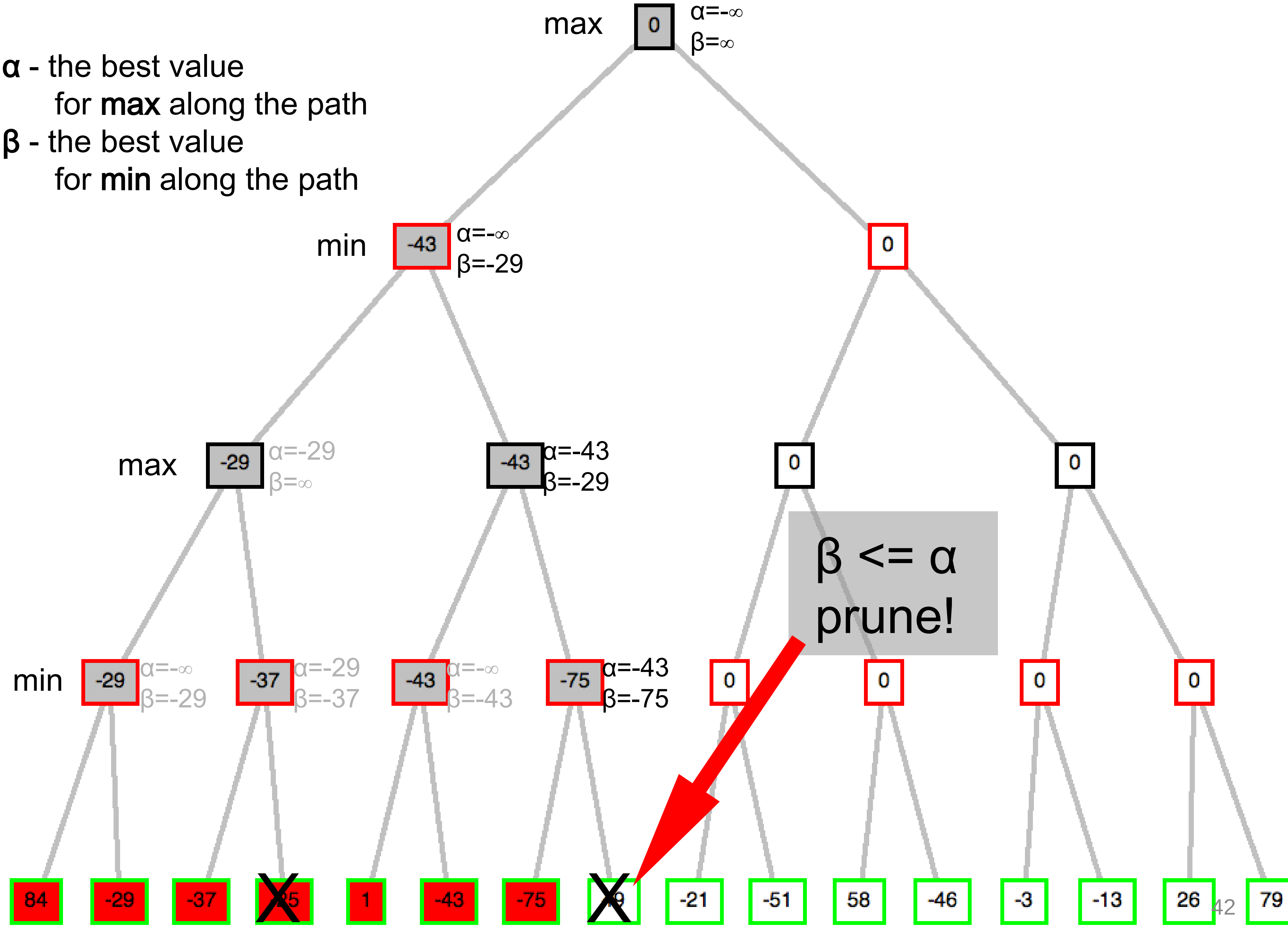

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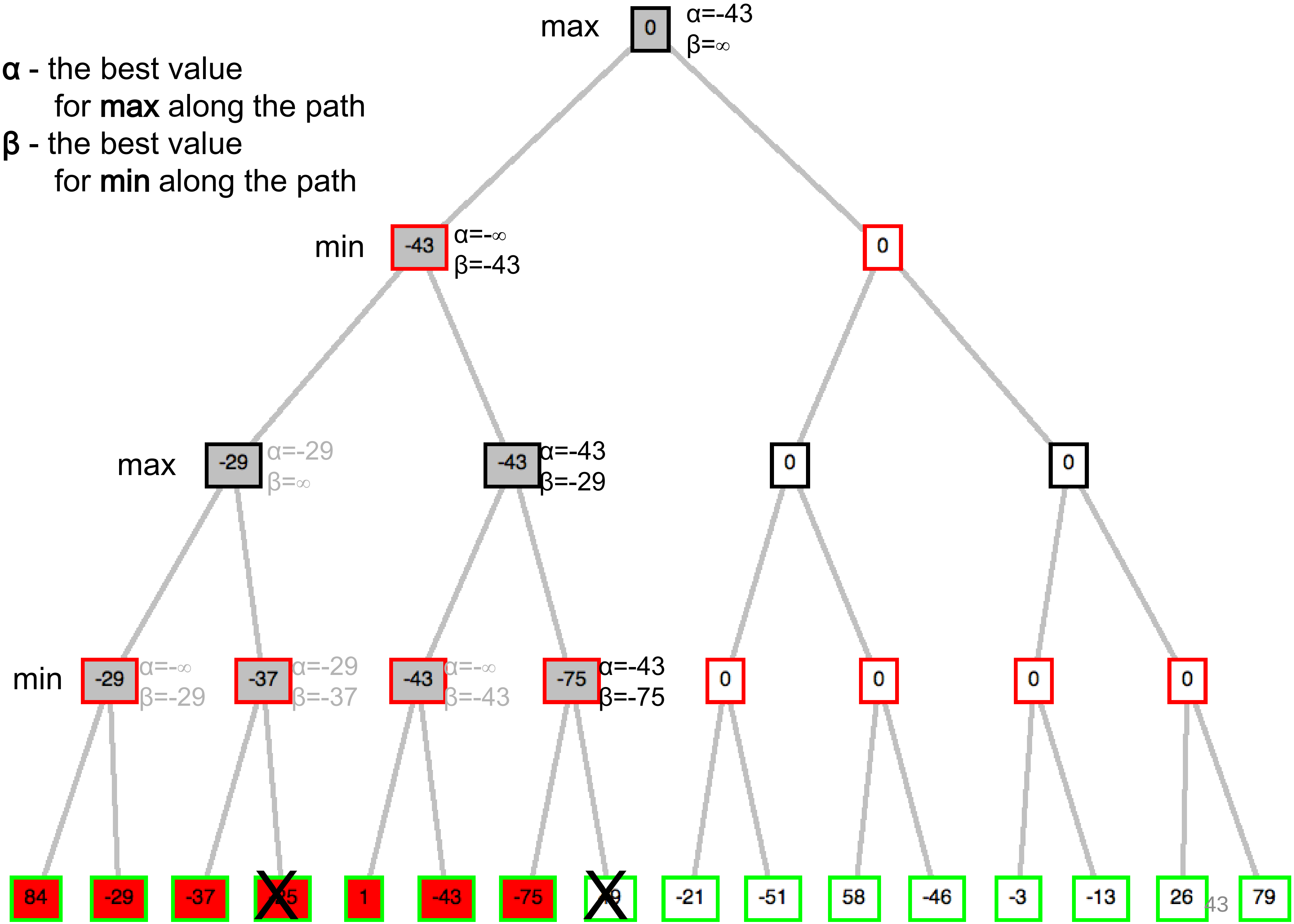

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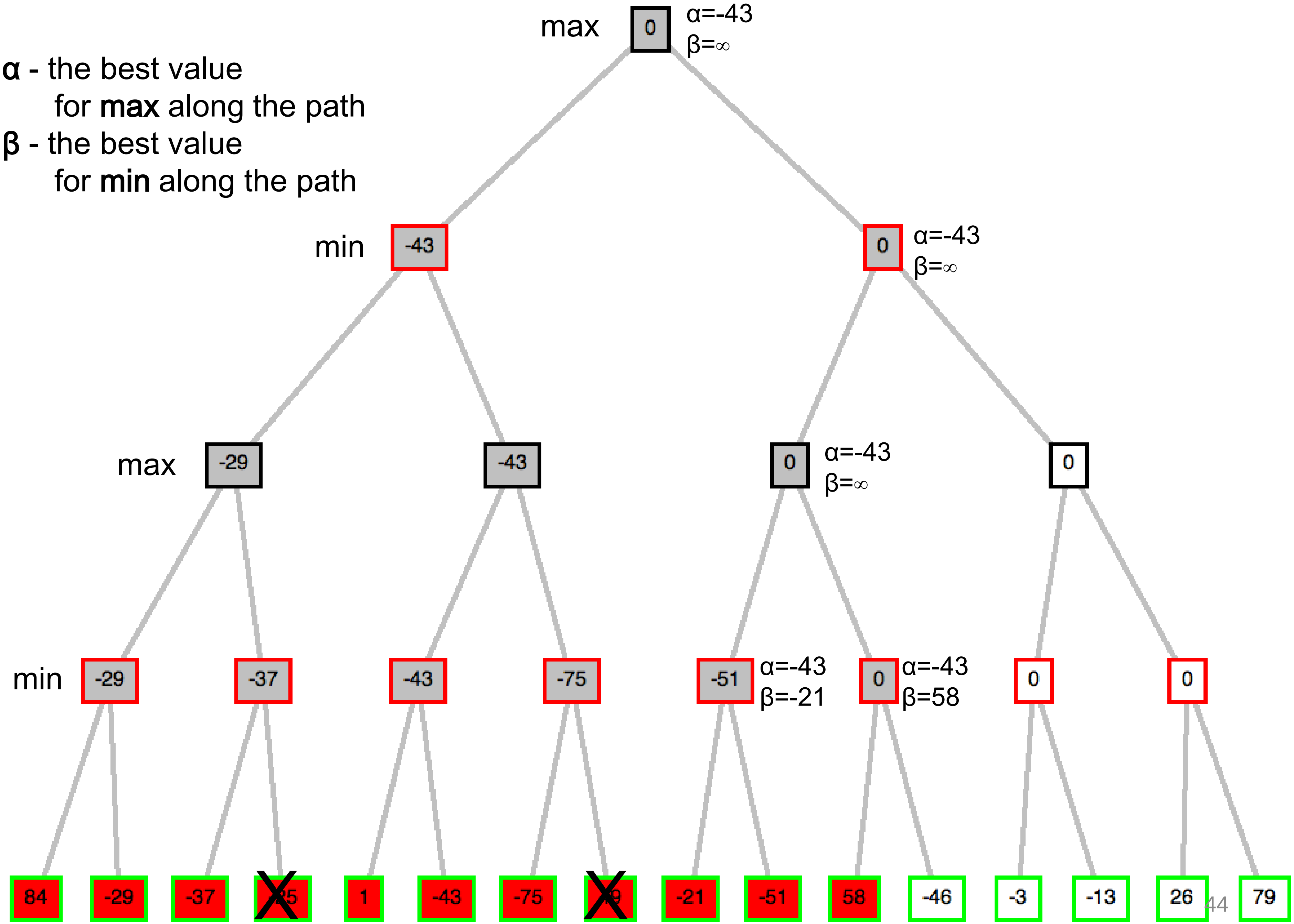

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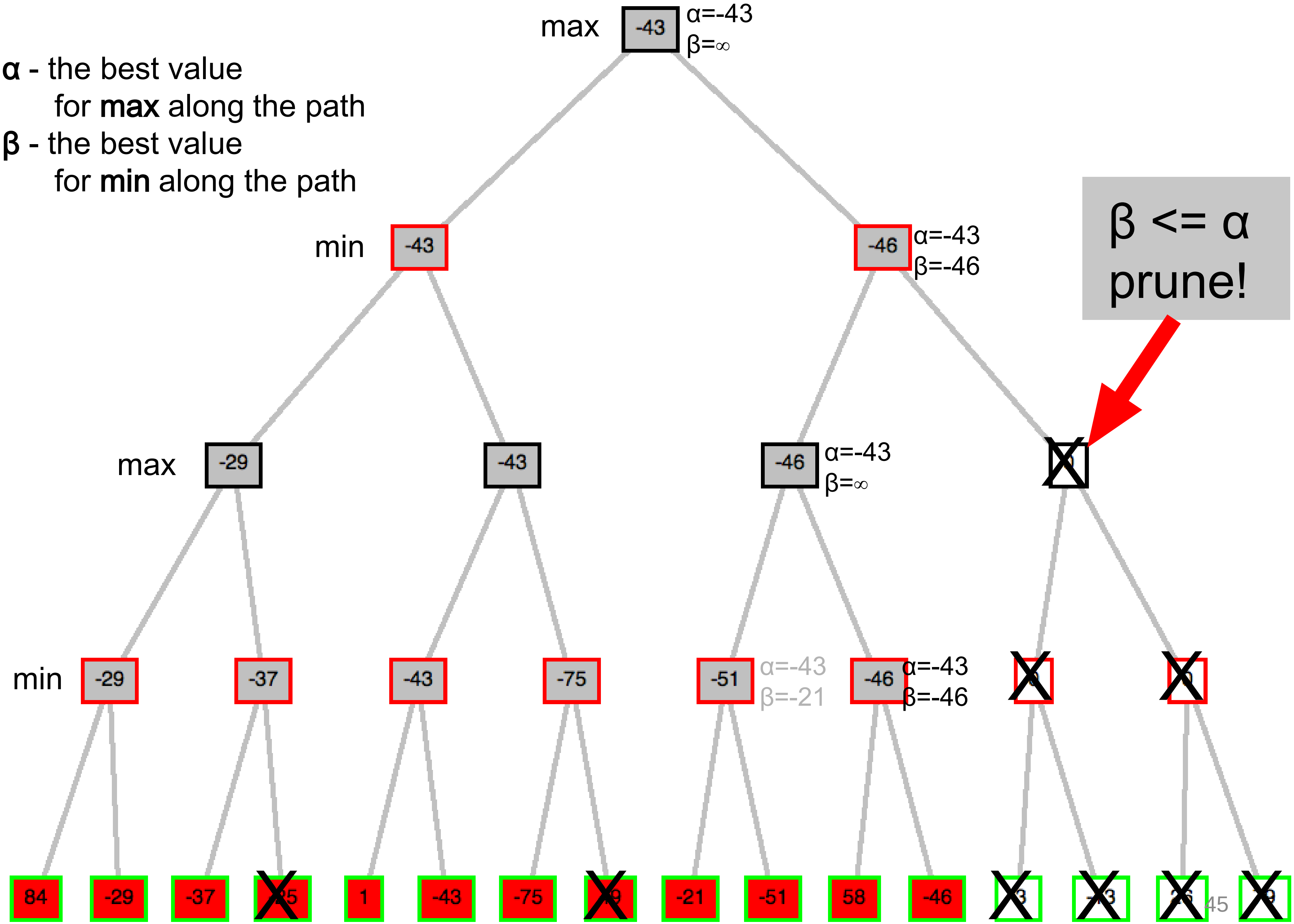


# Bad and Good Cases for Alpha-Beta Pruning

- Bad: Worst moves encountered first

```
                        4                       MAX
          +---------------+---------------+
          2               3               4          MIN
     +----+----+     +----+----+     +----+----+
     6    4    2     7    5    3     8    6    4      MAX
     +--+ +--+ +--+ +-+-+ +--+ +--+ +--+ +--+ +--+--+
     6  5 4  3 2  1 1 3 7 4  5 2  3 8  2 1 6 1  2  4
```

- Good: Good moves ordered first

```
                        4                       MAX
          +---------------+---------------+
          4               3               2          MIN
     +----+----+     +----+----+     +----+----+
     4    6    8     3    x    x     2    x    x      MAX
    +--+ +--+ +--+ +--+             +-+-+
    4  2 6  x 8  x 3  2             1 2 1
```

- If we can order moves, we can get more benefit from alpha-beta pruning

# Properties of α-β

- Pruning does not affect final result. This means that it gets the exact same result as does full minimax.

- Good move ordering improves effectiveness of pruning

- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search

- A simple example of reasoning about ‘which computations are relevant’ (a form of metareasoning)

# Node Ordering

Iterative deepening search

Use evaluations of the previous search for order

Also helps in returning a move in given time

# Good Enough?

- Chess:
  - branching factor $b \approx 35$
  - game length $m \approx 100$
  - search space $b^{m/2} \approx 35^{50} \approx 10^{77}$
- The Universe:
  - number of atoms $\approx 10^{78}$
  - age $\approx 10^{18}$ seconds
  - $10^{8}$ moves/sec x $10^{78}$ x $10^{18} = 10^{104}$

**The universe can play chess - can we?**

# Cutting off Search

*MinimaxCutoff* is identical to *MinimaxValue* except

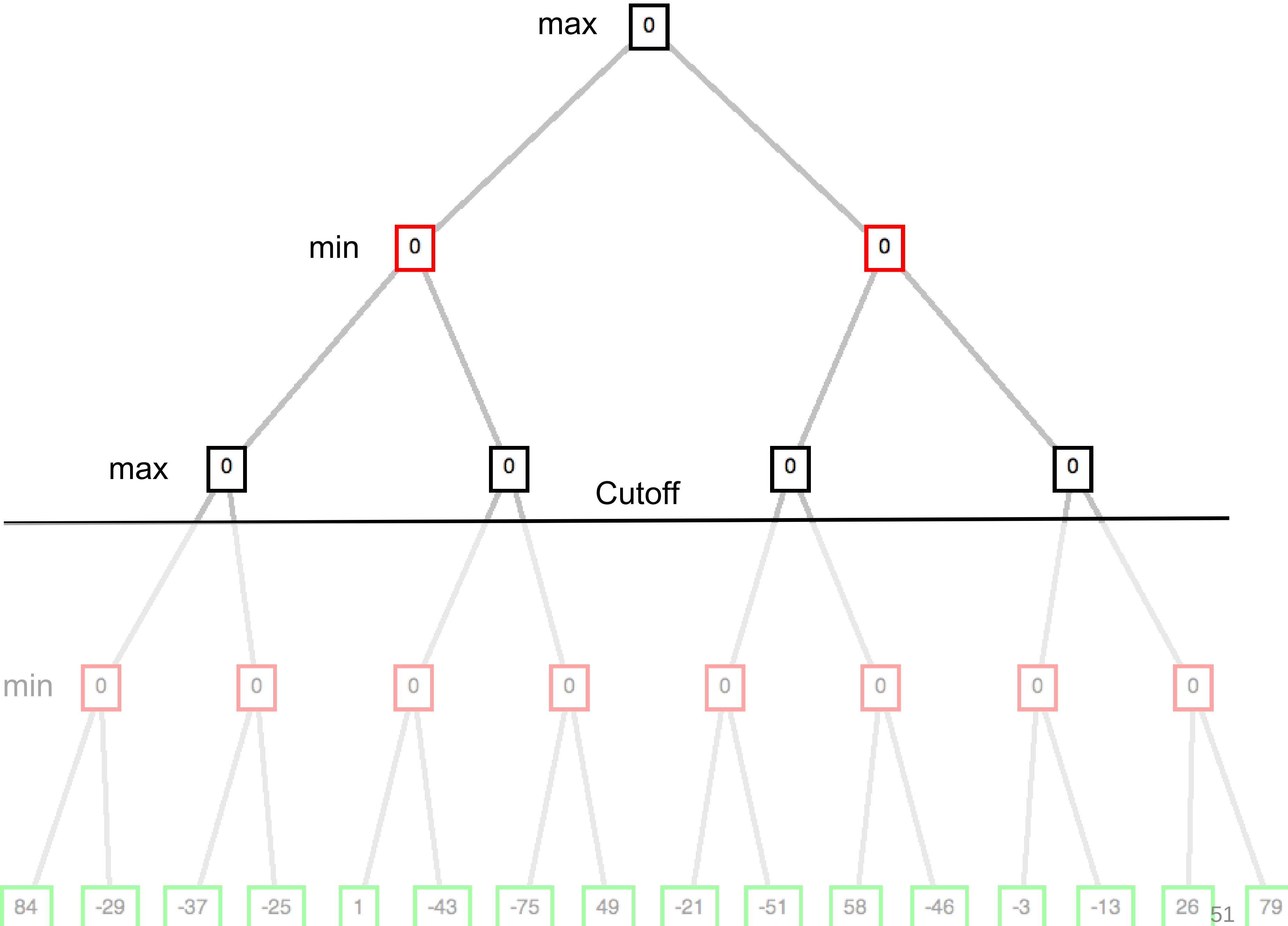
1. *Terminal?* is replaced by *Cutoff?*
2. *Utility* is replaced by *Eval*

Does it work in practice?

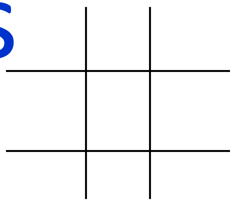
$b^m = 10^6$, $b=35 \rightarrow m=4$

4-ply lookahead is a hopeless chess player!

– 4-ply ≈ human novice
– 8-ply ≈ typical PC, human master
– 12-ply ≈ Deep Blue, Kasparov


max
0
min
0
0
max
0
0
0
0
Cutoff
min
0
0
0
0
0
0
0
0
84
-29
-37
-25
1
-43
-75
49
-21
-51
58
-46
-3
-13
26
79

# Evaluation Functions
# Tic Tac Toe



- Let p be a position in the game
- Define the utility function f(p) by
  - f(p) =
    - largest positive number if p is a win for computer
    - smallest negative number if p is a win for opponent
    - RCDC – RCDO
  - where RCDC is number of rows, columns and diagonals in which computer could still win
  - and RCDO is number of rows, columns and diagonals in which opponent could still win.

# Sample Evaluations

- X = Computer; O = Opponent

| | | |
|---|---|---|
| | O | |
| | X | |
| | | |

| | | |
|---|---|---|
| O | O | X |
| X | X | |
| | | |

| | X | O |
|---|---|---|
| rows | | |
| cols | | |
| diags | | |

| | X | O |
|---|---|---|
| rows | | |
| cols | | |
| diags | | |

# Evaluation functions

- For chess/checkers, typically linear weighted sum of features

$$Eval(s) = w_1\ f_1(s) + w_2\ f_2(s) + \ldots + w_n\ f_n(s)$$

e.g., $w_1 = 9$ with

$f_1(s)$ = (number of white queens) – (number of black queens), etc.

# Example: Samuel's Checker-Playing Program

- It uses a linear evaluation function

  $f(n) = a_1x_1(n) + a_2x_2(n) + \ldots + a_mx_m(n)$

  For example: $f = 6K + 4M + U$

  – K = King Advantage

  – M = Man Advantage

  – U = Undenied Mobility Advantage (number of moves that Max where Min has no jump moves)

# Samuel's Checker Player

- In learning mode
  - Computer acts as 2 players: A and B
  - A adjusts its coefficients after every move
  - B uses the static utility function
  - If A wins, its function is given to B

# Samuel's Checker Player

- How does A change its function?
  1. Coefficent replacement

$\Delta$(node ) = backed-up value(node) – initial value(node)

if $\Delta > 0$ then terms that contributed **positively** are given more weight and terms that contributed negatively get less weight

if $\Delta < 0$ then terms that contributed **negatively** are given more weight and terms that contributed positively get less weight

# Samuel’s Checker Player

- How does A change its function?

  2. Term Replacement

     38 terms altogether

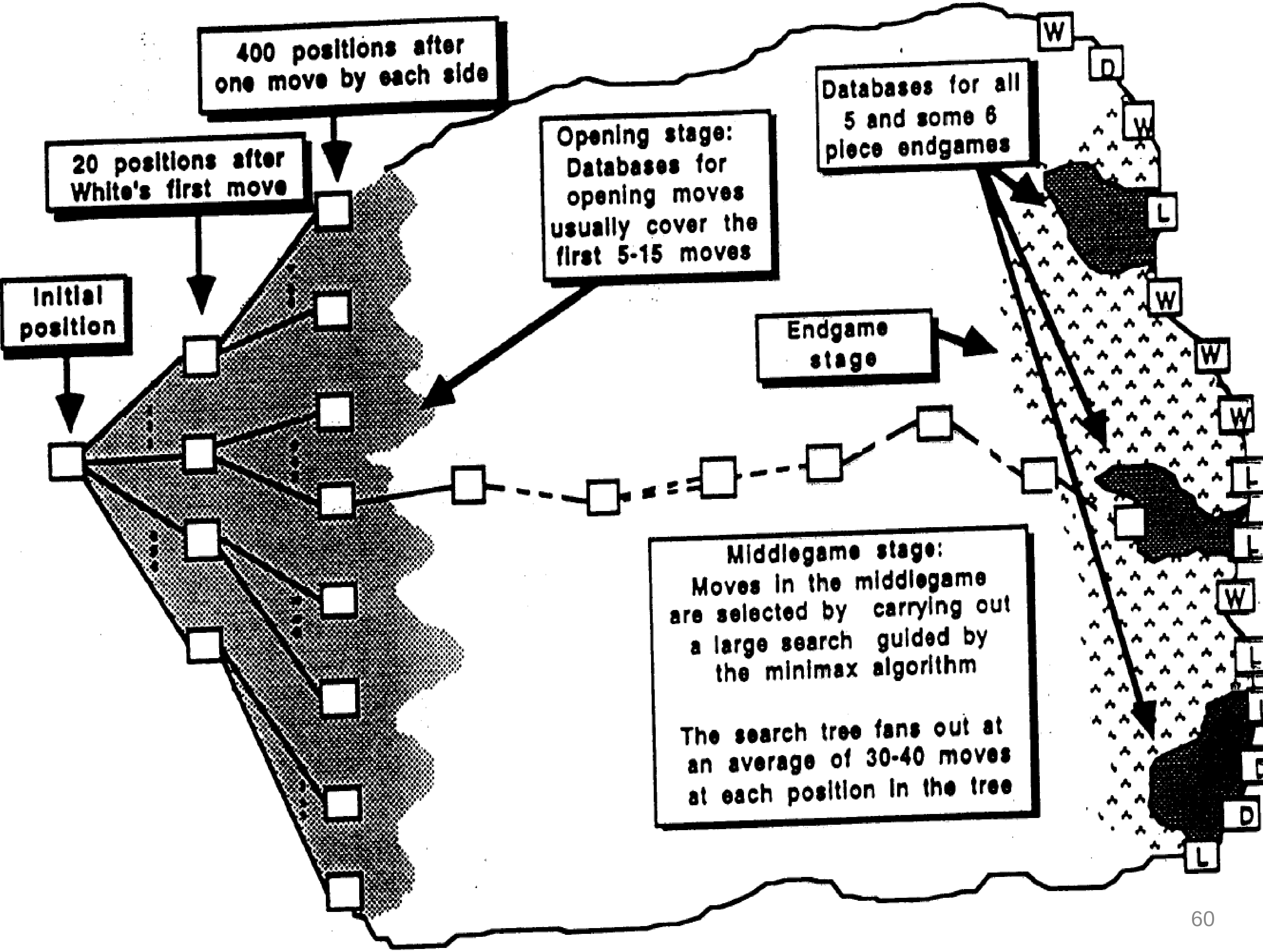
     16 used in the utility function at any one time

     Terms that **consistently correlate low** with the function value are removed and added to the end of the term queue.

     They are replaced by terms from the front of the term queue.

# Chess: Rich history of cumulative ideas

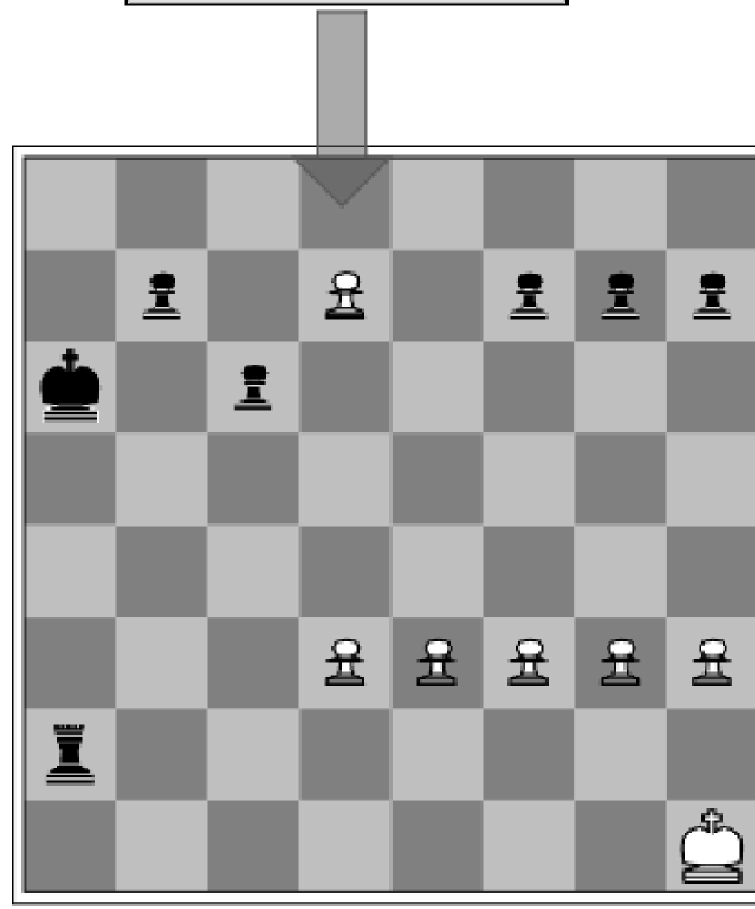
- Minimax search, evaluation function learning (1950).
- Alpha-Beta search (1966).
- Transposition Tables (1967).
- Iterative deepening DFS (1975).
- End game data bases ,singular extensions(1977, 1980)
- Parallel search and evaluation(1983 ,1985)
- Circuitry (1987)


400 positions after one move by each side
20 positions after White's first move
Initial position
Opening stage: Databases for opening moves usually cover the first 5-15 moves
Databases for all 5 and some 6 piece endgames
Endgame stage
Middlegame stage: Moves in the middlegame are selected by carrying out a large search guided by the minimax algorithm
The search tree fans out at an average of 30-40 moves at each position in the tree
W
D
W
L
W
W
W
L
L
W
L
L
D
D
L

# Problem with fixed depth Searches

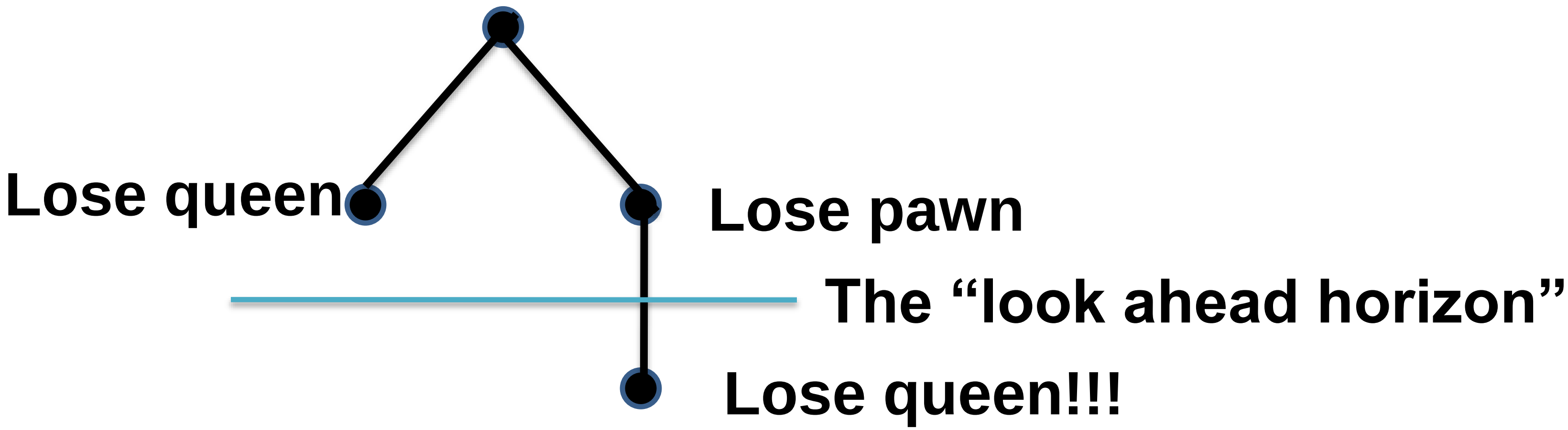
if we only search n moves ahead, it may be possible that the catastrophy can be delayed by a sequence of moves that do not make any progress

also works in other direction (good moves may not be found)




Black to move

# Problems with a fixed ply: The Horizon Effect




- Inevitable losses are postponed
- Unachievable goals appear achievable
- Short-term gains mask unavoidable consequences (traps)

# Solutions

- How to counter the horizon effect
  - Feedover
    - Do not cut off search at non-quiescent board positions (dynamic positions)
    - Example, king in danger
    - Keep searching down that path until reach quiescent (stable) nodes
  - Secondary Search
    - Search further down selected path to ensure this is the best move
  - Progressive Deepening
    - Search one ply, then two ply, etc., until run out of time
    - Similar to IDS

# Quiescence Search

This involves searching past the terminal search nodes (depth of 0) and testing all the non-quiescent or 'violent' moves until the situation becomes calm, and only then apply the evaluator.

Enables programs to detect long capture sequences and calculate whether or not they are worth initiating.

Expand searches to avoid evaluating a position where tactical disruption is in progress.

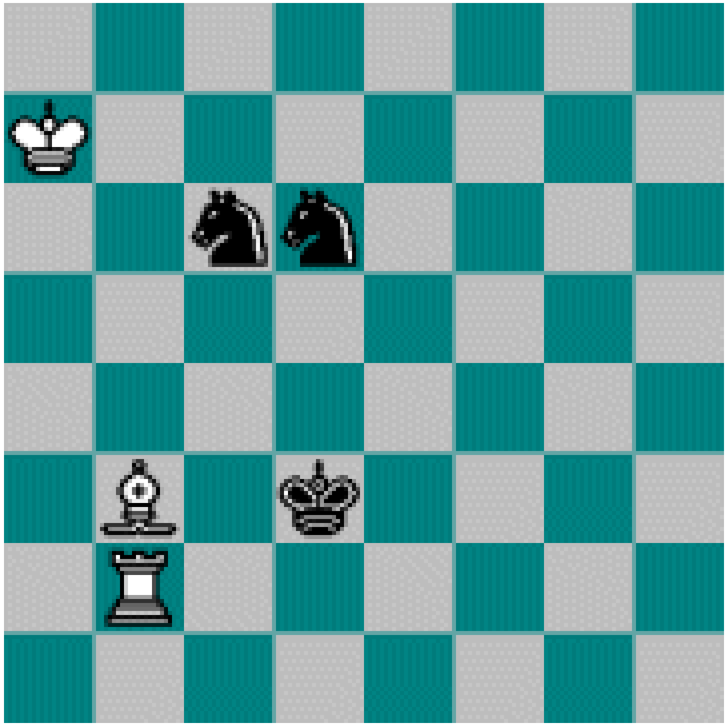
# Additional Refinements

- Probabilistic Cut: cut branches probabilistically based on shallow search and global depth-level statistics (forward pruning)
- Openings/Endgames: for some parts of the game (especially initial and end moves), keep a catalog of best moves to make.
- Singular Extensions: find obviously good moves and try them at cutoff.

# End-Game Databases

- Ken Thompson - all 5 piece end-games
- Lewis Stiller - all 6 piece end-games
  - Refuted common chess wisdom: many positions thought to be ties were really forced wins -- 90% for white
  - Is perfect chess a win for white?

# The MONSTER



White wins in 255 moves

(Stiller, 1991)

# Deterministic Games in Practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a precomputed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions. Checkers is now solved!

- Chess: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic!

- Othello: human champions refuse to compete against computers, who are too good.

- Go: human champions refuse to compete against computers, who are too bad. In Go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves, along with aggressive pruning.

# Game of Go

human champions refuse to compete against computers, because software is too bad.

| | Chess | Go |
|---|---|---|
| Size of board | 8 x 8 | 19 x 19 |
| Average no. of moves per game | 100 | 300 |
| Avg branching factor per turn | 35 | 235 |
| Additional complexity | | Players can pass |

# Recent Successes in Go

- MoGo defeated a human expert in 9x9 Go
- Still far away from 19x19 Go.

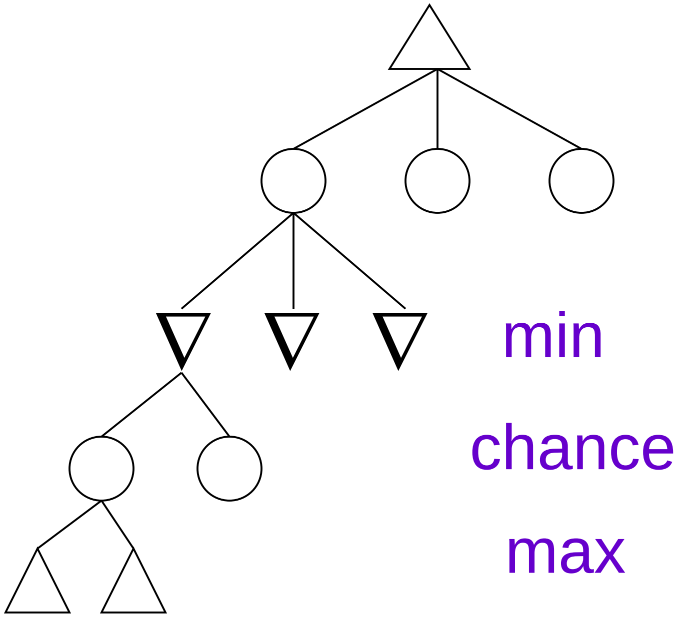
- Hot area of research
- Leading to development of novel techniques
  - Monte Carlo tree search (UCT)

# Other Games

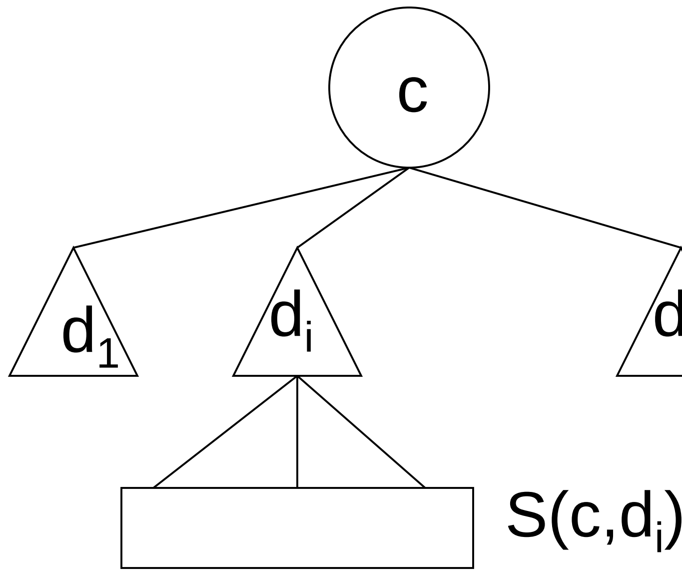
| | deterministic | chance |
|---|---|---|
| perfect information | chess, checkers, go, othello | backgammon, monopoly |
| imperfect information | stratego | bridge, poker, scrabble |

# Games of Chance

- What about games that involve chance, such as
  - rolling dice
  - picking a card
- Use three kinds of nodes:
  - max nodes
  - min nodes
  - chance nodes



# Games of Chance
# Expectiminimax

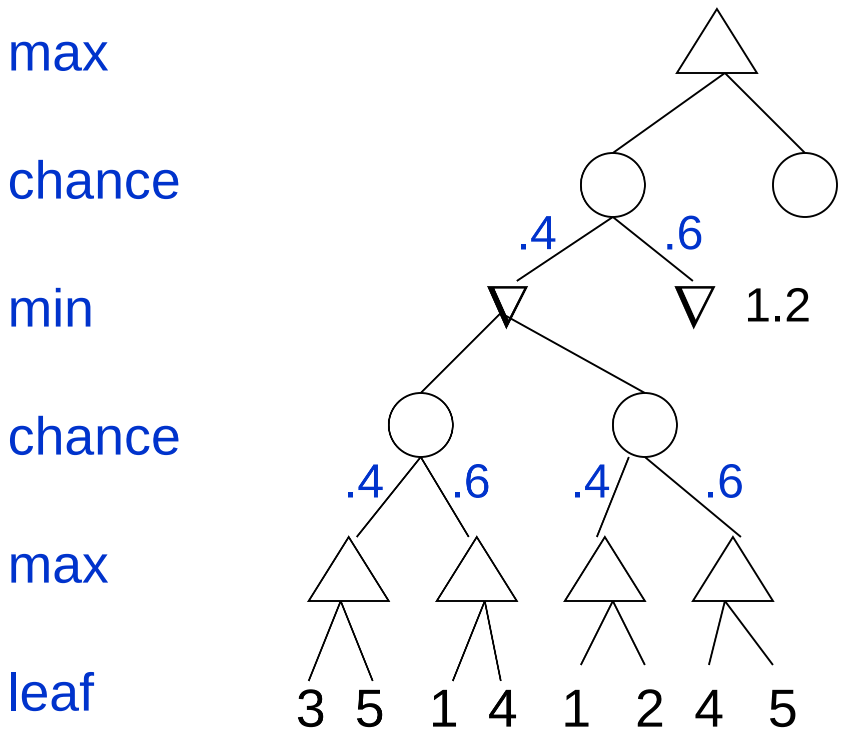



chance node with max children

$$\text{expectimax}(c) = \sum_{i} P(d_i) \max_{s \text{ in } S(c,d_i)} (\text{backed-up-value}(s))$$

$$\text{expectimin}(c') = \sum_{i} P(d_i) \min_{s \text{ in } S(c,d_i)} (\text{backed-up-value}(s))$$

# Example Tree with Chance



# Complexity

- Instead of $O(b^m)$, it is $O(b^m n^m)$ where $n$ is the number of chance outcomes.
- Since the complexity is higher (both time and space), we cannot search as deeply.
- Pruning algorithms may be applied.

# Imperfect Information

- E.g. card games, where opponents' initial cards unknown
- Idea: For all deals consistent with what you can see
  - compute the minimax value of available actions for each of possible deals
  - compute the expected value over all deals

# Status of AI Game Players

- Tic Tac Toe
  - Tied for best player in world
- Othello
  - Computer better than any human
  - Human champions now refuse to play computer
- Scrabble
  - Maven beat world champions Joel Sherman and Matt Graham
- Backgammon
  - 1992, Tesauro combines 3-ply search & neural networks (with 160 hidden units) yielding top-3 player
- Bridge
  - Gib ranked among top players in the world
- Poker
  - 2015, Heads-up limit hold'em poker is solved
- Checkers
  - 1994, Chinook ended 40-year reign of human champion Marion Tinsley
- Chess
  - 1997, Deep Blue beat human champion Gary Kasparov in six-game match
  - Deep Blue searches 200M positions/second, up to 40 ply
  - Now looking at other applications (molecular dynamics, drug synthesis)
- Go
  - 2008, MoGo running on 25 nodes (800 cores) beat Myungwan Kim
  - $2M prize available for first computer program to defeat a top player

# Summary

- Games are fun to work on!
- They illustrate several important points about AI.
- Perfection is unattainable → must approximate.
- Game playing programs have shown the world what AI can do.